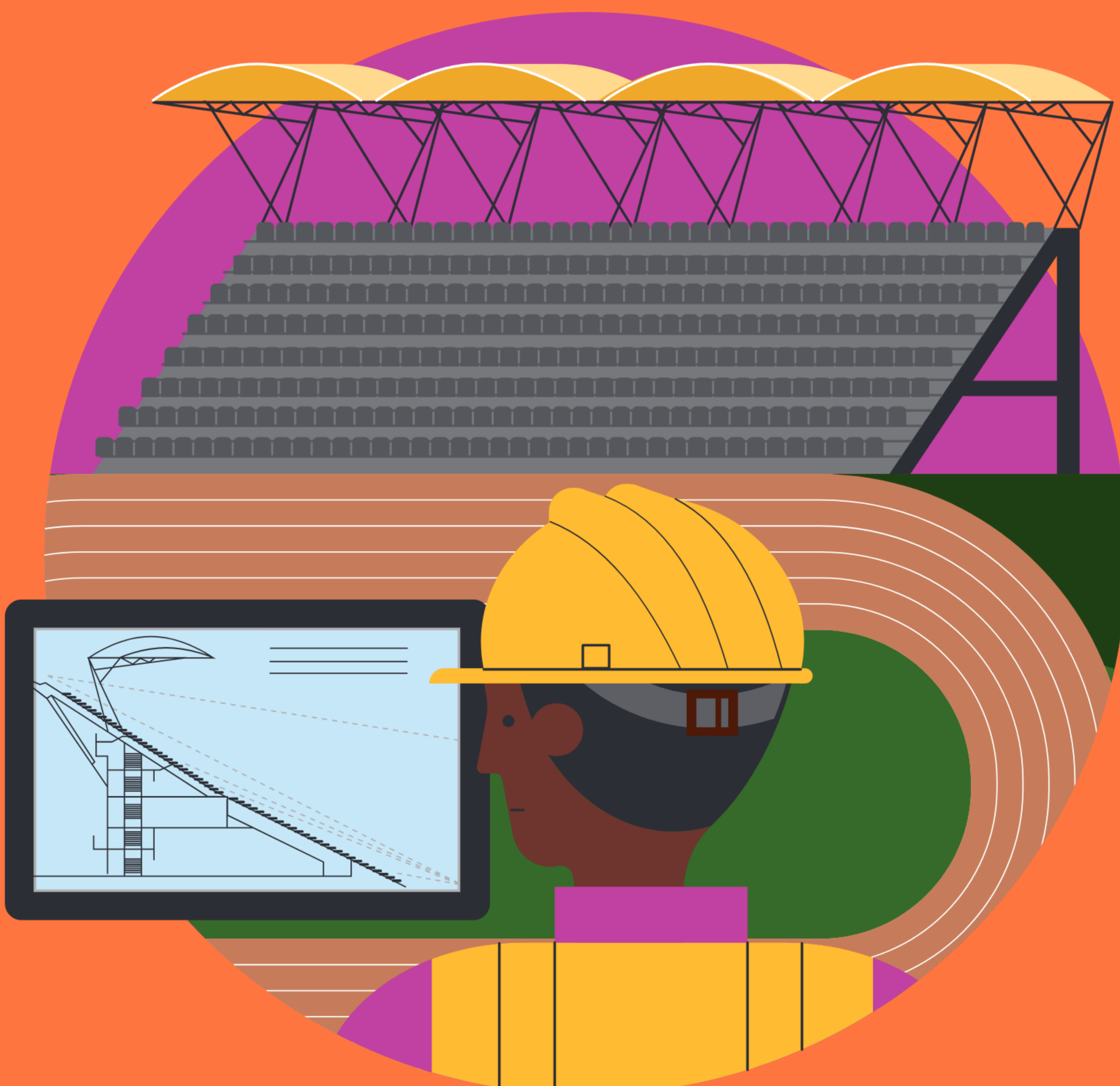
Oxford International Resources

9

Maths

Student Book



Lower Secondary

OXFORD

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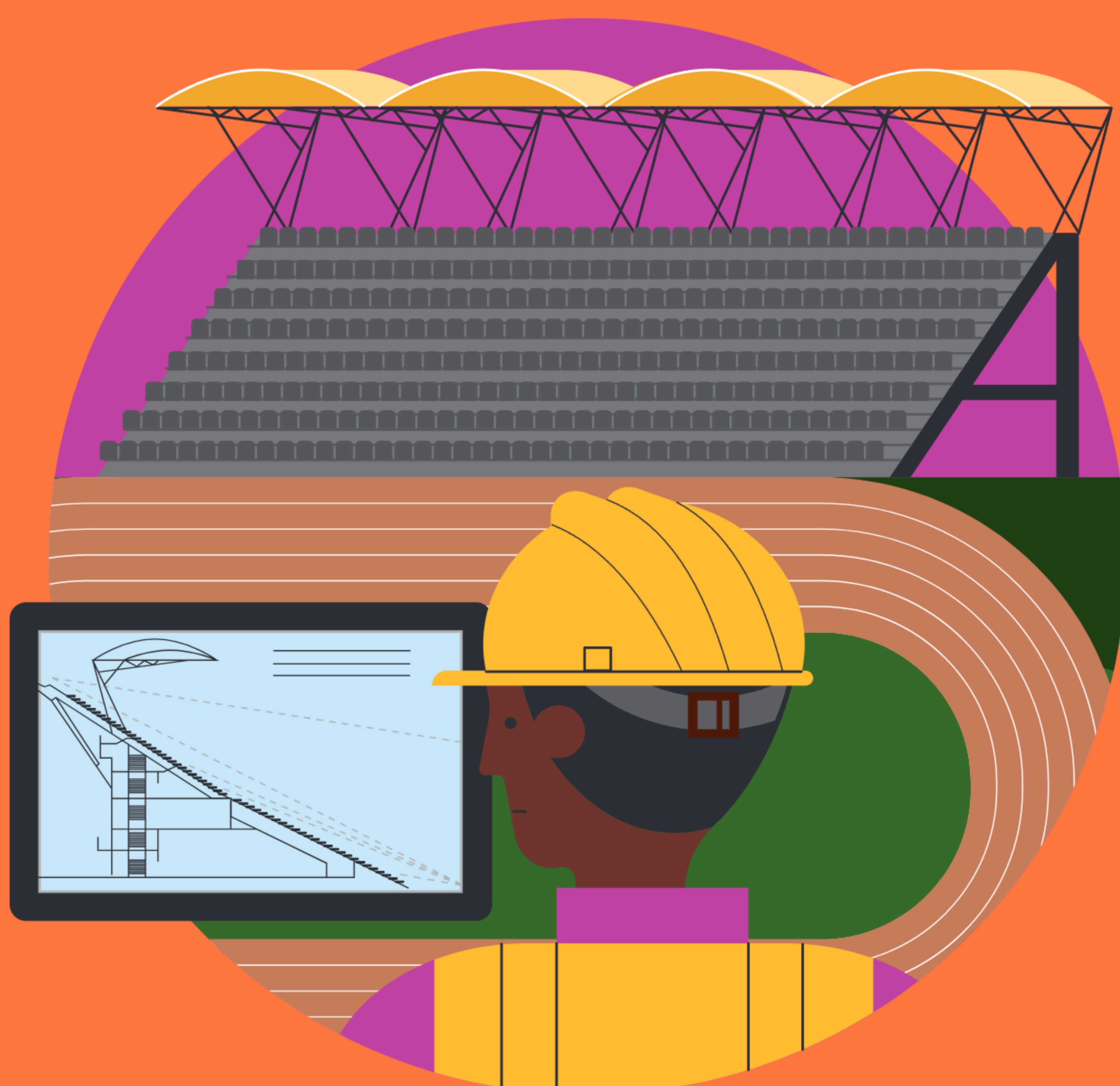

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Contents

How to use this book

Each topic begins with a set of learning objectives. These tell you what you will be able to do by the end of the lesson.

Key idea

The key idea summarizes the main points of each topic in a few sentences.

Key words

The key words for each topic are highlighted in **bold** in the text. They are also included in order of appearance in this box. You can also find them in the Glossary at the back of your Student Book.

Fluency questions

These questions check your understanding of a topic before moving on to the next.

Stretch zone

This icon shows you where you will need to think more deeply. It is OK if you find the 'stretch zone' questions difficult. Instead of giving up, keep thinking and trying. You will get there – doing challenging work is the exercise your brain needs!

Welcome to your Student Book

This introduction shows you all the different features *Oxford International Maths* has to support you on your journey through Lower Secondary Maths.

Being a mathematician (someone who studies maths) is great fun. As you work through this Student Book, you will learn how to work mathematically and become confident (or even more confident!) in your maths skills.

Each chapter in this book covers a few topics. With plenty of worked examples and practice questions, you will study these topics for a few weeks to make sure you have time to learn them properly.

Literacy skills

These boxes tell you more about the history and use of key vocabulary to put the new words you learn in context.

Stretch zone

These boxes suggest ideas for how to take your learning further and discover something more than what is in the pages of your Student Book.

Calculator skills

These boxes help you get to know your calculator and use it effectively.

Some of the topics in this book would be almost impossible without a calculator. In other topics, your calculator will be useful for doing lots of calculations quickly or for checking answers.

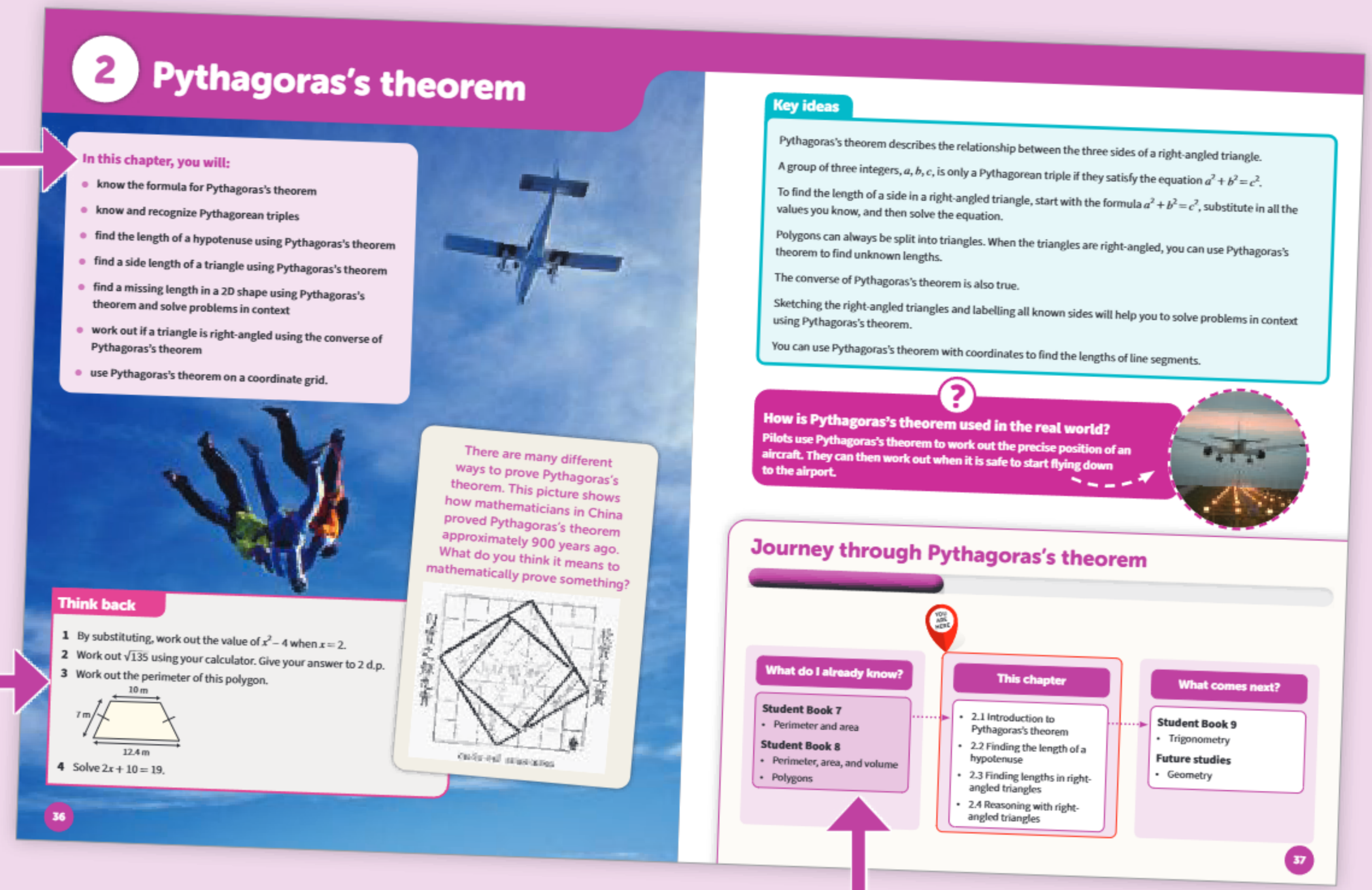
Do not use your calculator for all your maths. You should still be confident to carry out calculations, both in your head and by using written methods. Your teacher will tell you when to use a calculator and when they want you to work out a maths problem without one.

Chapter opener

Each chapter begins with an introduction. This reminds you what you already know and shows you what is coming up in the chapter.

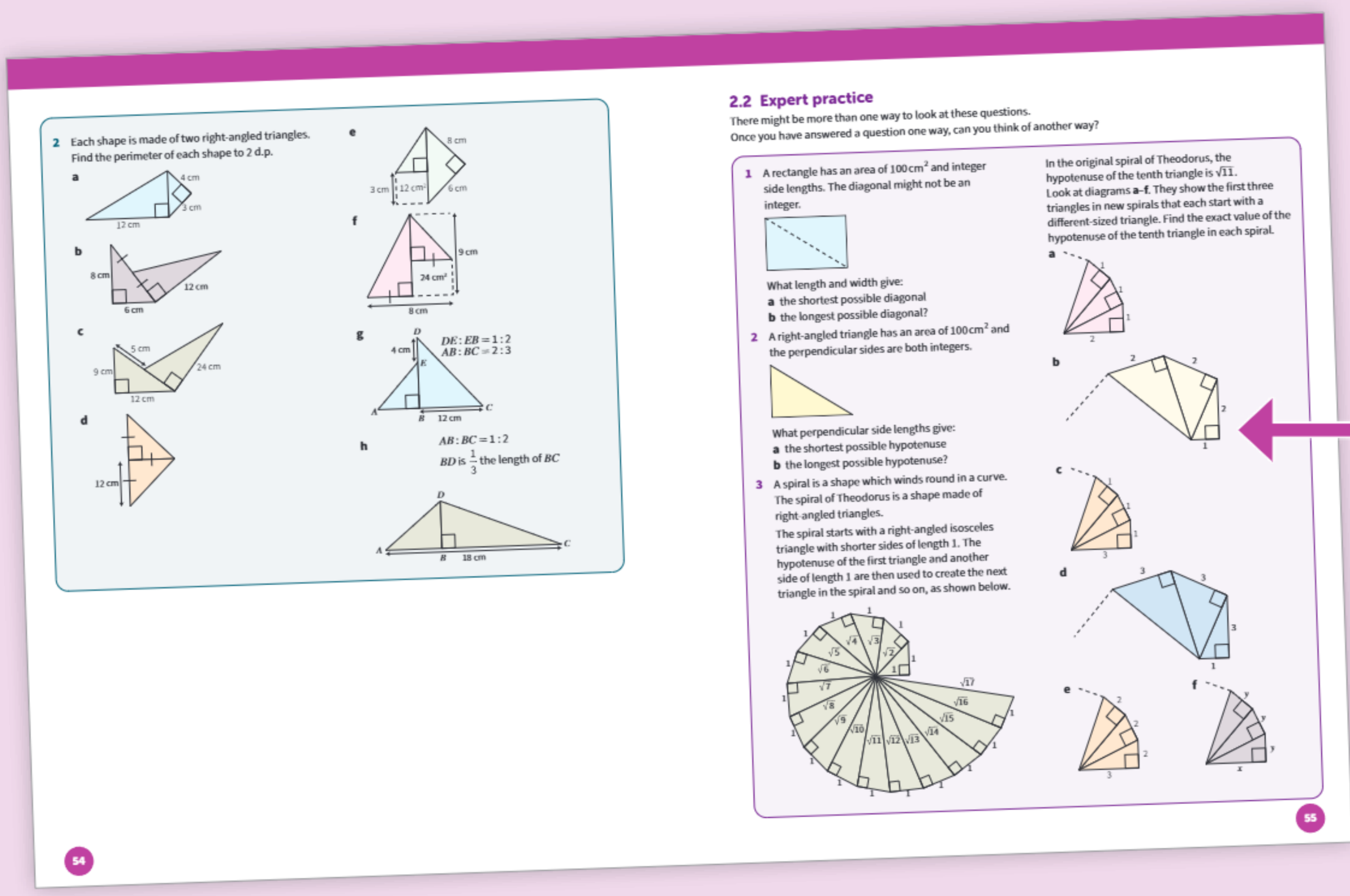
Think back

These quick questions help you recall the maths you already know. To be successful with a new topic in maths, you need to build on your existing knowledge and fill in any gaps before carrying on.



Chapter map

This map shows clearly what maths you already know, the new topics you will study in this chapter, and the next steps in your maths learning.



Become an expert at each topic

There are three different ways to practise maths at the end of every section:

1 Intelligent practice

2 Which method?

3 Expert practice

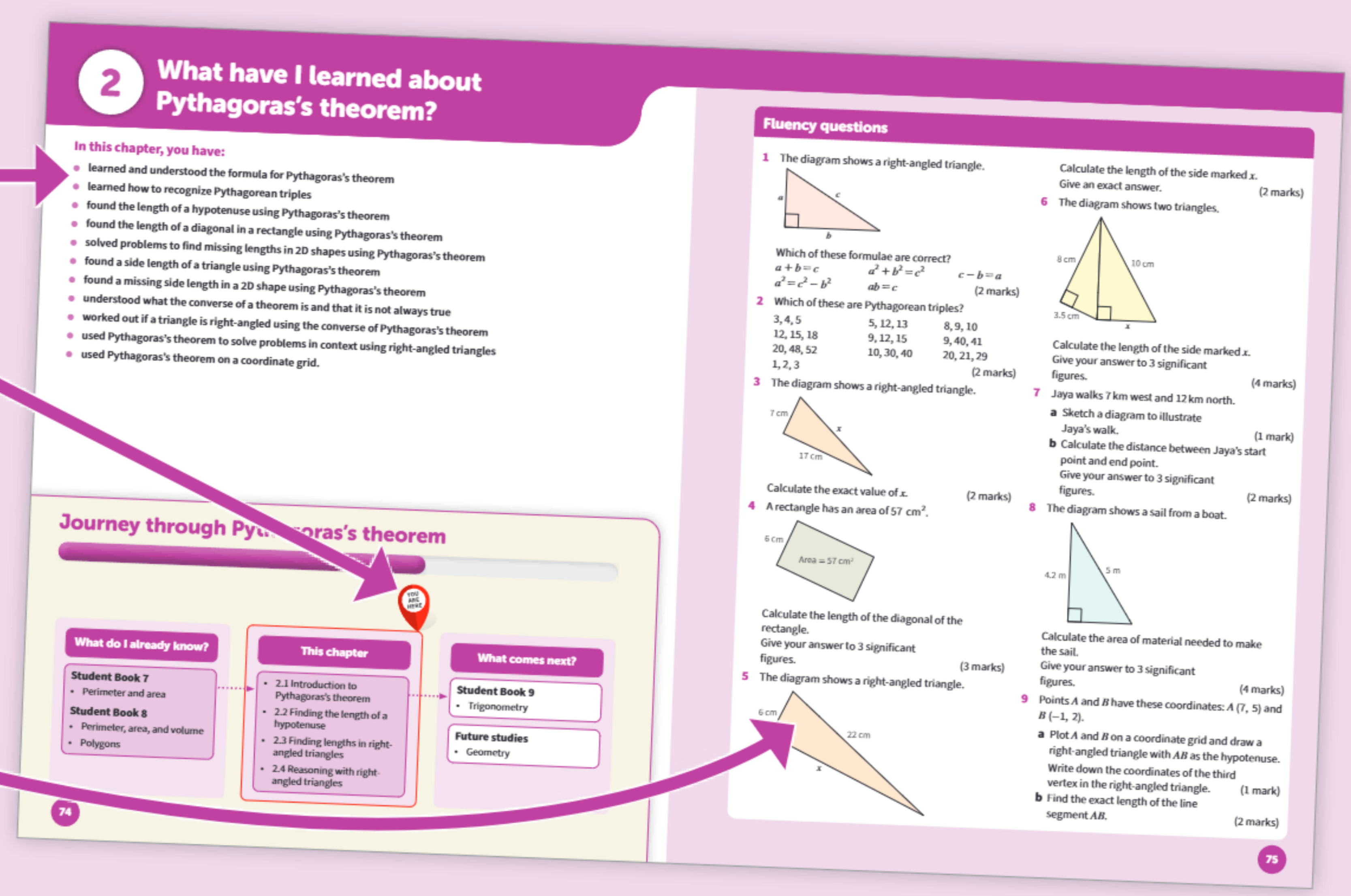
Each exercise works in a particular way to help your brain make connections, remember the topic, and recognize when to use it.

Chapter summary

This summarizes what you have learned so far and shows your progress through the unit.

Fluency questions

You can use these exam-style questions to test how well you know the topics in the chapter.



How to use example-problem pairs

Example-problem pair (EPP) grids are a special type of worked example that help you understand what you are doing at each step and why.

There are lots of different ways you can use the example-problem pairs. Here is one possible way:

1 Start with the **Worked example** on the left.

2 Think about each line of working using the questions in the **Thinking** column.

Worked example	Thinking	Your turn!
Write $8 \times 8 \times 8 \times 8 \times 8$ in index notation	*Are we repeatedly multiplying the same number?* Yes, so I need to count how many times 8 has been multiplied	**Write $9 \times 9 \times 9 \times 9 \times 9 \times 9$ in index notation**
There are 5 copies of the number 8 being multiplied	*How many times are we multiplying that number?* I counted the number of 8s that were written	
8 is the base number and 5 is the exponent	*Which value is the exponent and which is the base number?* The number being multiplied is the base number and the number of copies is the exponent	
So $8 \times 8 \times 8 \times 8 \times 8 = 8^5$		

3 Try to predict what the next line of working will be before you look at it.

4 Once you have thought about the example on the left, move to the **Your turn!** question on the right. This question will be very similar to the example you have just studied.

You can use the same thinking ideas to answer this question one step at a time.

The questions in the **Thinking** column help you think more generally about the example, so that you understand how to think about a different question.

The **Your turn!** question lets you apply the new idea with some support, so that you can be confident in what you need to do before you move on to the **Fluency** questions.

How to use Reflect, Expect, Check, Explain

For the **Intelligent practice** questions, use the Reflect, Expect, Check, Explain (RECE) method. This means you think about the question you are about to do, compare it to the one you have just done, and predict how the answer will be different. This is a great technique for developing your reasoning skills – plus it gives you an opportunity to discuss things with your partner, or as a class, which helps you become more confident talking about maths.

1. **Reflect**: Read the question. What has changed in this question compared to the previous one? What has stayed the same?
2. **Expect**: Using your reflection from Step 1 and the answer to the previous question, what do you think the answer will be? Can you explain why you think that?
3. **Check** your expectation by carrying out the usual method to answer the question.
4. **Explain**: Was your expectation in Step 2 correct? If the answer surprises you, can you explain why? If the answer is what you expected, how could you explain your reasoning to someone else? If you were not able to make a prediction in Step 2, can you explain the relationship now?

Look at the example below.

Question 2a

Is 72 a multiple of 9? How do you know?

I recognize 72 from times tables: 72 = 9 × 8. So, yes, 72 is a multiple of 9.

Or, I could use the divisibility rule for 9.

The digit sum is 7 + 2 = 9 and 9 is a multiple of 9, so 72 must be a multiple of 9.

Question 2b

Is 720 a multiple of 9? How do you know?

Reflect: *This question is like* **2a** *because it deals with multiples of 9. The number is just larger.*

Expect: *720 is 10 × 72. So, 720 must be a multiple of 9 too.*

Check: *Using the divisibility rule for 9, the digit sum 7 + 2 + 0 is 9 and 9 is a multiple of 9.*

Explain: *I was right! 720 is a multiple of 9 because the digit sum is a multiple of 9 and 72 = 9 × 8, so 720 = 9 × 80.*

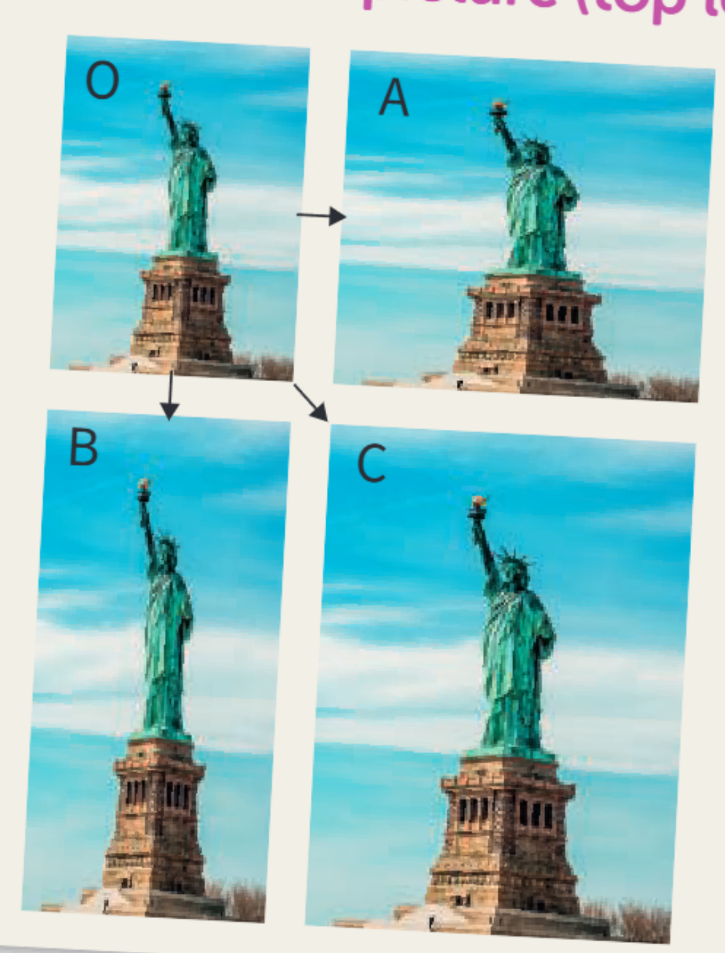
You can also use this method when you are working through the **Your turn!** question in an example-problem pair.

1 Similarity and congruence

In this chapter, you will:

- label sides and angles of triangles and other polygons
- describe triangles with equal angles and right-angled triangles
- identify similar shapes
- find missing angles and missing side lengths in similar shapes
- use and describe properties of congruent shapes
- identify congruent triangles using congruence tests.

Jen tries changing the size of a small image of the Statue of Liberty in computer software. Depending on how she clicks and drags from the edge of the image, she gets different results. Three examples are shown. Which of the three versions A–C looks most like the original version of the picture (top left)?



Think back

1 Copy and complete these tables of pairs of numbers that satisfy the same multiplicative relationship.

a

8	56
10	

b

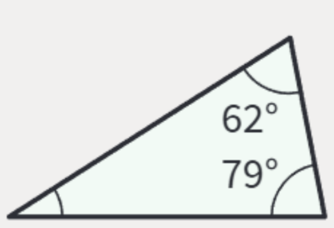
4.5	18
	24

c

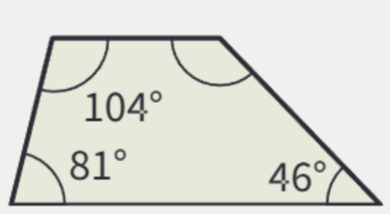
45	
105	21

2 Work out the size of the missing angles in these shapes.

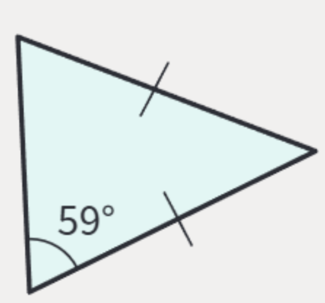
a



b



c



Key ideas

In any right-angled triangle, the hypotenuse is the longest side.

Two shapes are similar if their respective angles are the same and their respective sides are in the same proportion. Each shape is an enlargement of the other.

You can use the fact that angles in similar shapes are equal to find missing angles in some shapes.

In similar shapes, the multiplicative relationship is the same between each pair of matching sides.

Two shapes are congruent if they have the same respective angles and the same respective side lengths.

You can use the congruence tests to identify if two triangles are congruent.

How have similarity and congruence been used throughout history?

Thales of Miletus calculated the height of the Great Pyramid of Giza by measuring its shadow and multiplying, using his knowledge of similarity.

Journey through similarity and congruence

YOU ARE HERE

What do I already know?	This chapter	What comes next?
Student Book 7 • Transformations **Student Book 8** • Percentages and proportionality	• 1.1 Notation and naming • 1.2 Similarity • 1.3 Congruence	**Future studies** • Geometry

1.1 Notation and naming

1.1.1 Labelling polygons

After this topic, you will be able to:
- label sides and angles of triangles and other polygons.

Key idea

You label **points** and **vertices** with capital letters.

Key words

point, vertex (plural: vertices), line segment, polygon, triangle, acute angle, reflex angle, hexagon, quadrilateral, obtuse angle, sketch, right-angled triangle, area

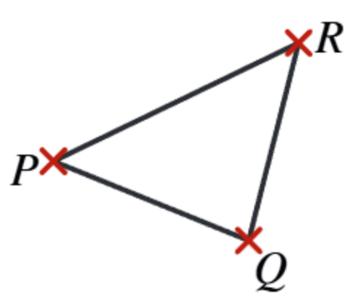
You label a point with a capital letter.

You say that this is 'point P'.

You name a **line segment** by using the two points at either end.

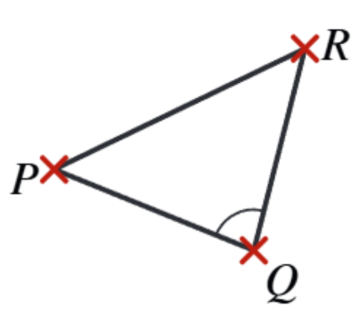
You say that this is 'line segment PQ'.

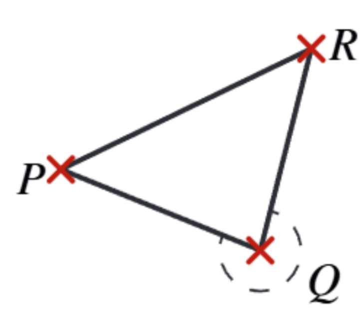
You describe a **polygon** using letters at each vertex.



You say that this is '**triangle** PQR'.

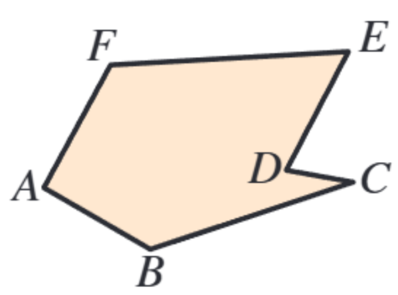
When you say 'angle PQR', you mean the angle shown by the arc. The position of the arc shows whether you are talking about the **acute angle** or the **reflex angle**.

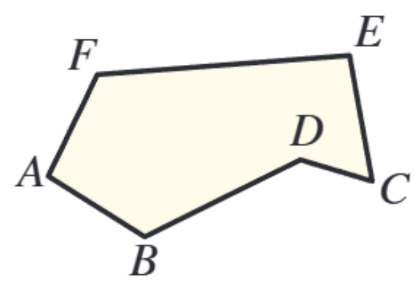




To write this angle, you can write 'angle PQR'. You can also write $\angle PQR$ or $P\hat{Q}R$.

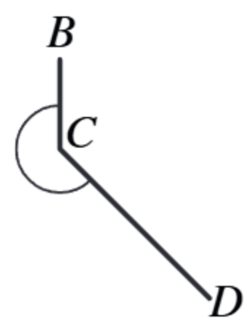
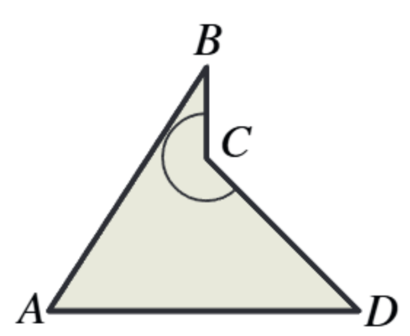
When you name a shape, you give the points in the order that they are joined.



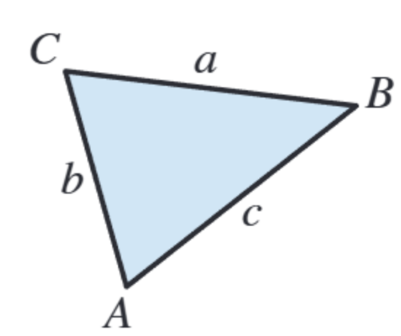


The first **hexagon** is named $ABCDEF$. The second hexagon is named $ABDCEF$. You write the letters in alphabetical order where possible, so the second hexagon is badly named!

Worked example	Thinking	Your turn!
Sketch a quadrilateral $ABCD$ where $\angle BCD$ is a reflex angle and side BC is the shortest side.	*Which property should we sketch first?* I will draw the shortest side first. I will then label this side.	**Sketch the quadrilateral $PQRS$ where $\angle RSP$ is an obtuse angle and side PQ is the shortest side.**

Worked example	Thinking	Your turn!
	*Which property should we* ***sketch*** *second?* I can now draw the reflex angle, so that the vertex C is the middle of the angle and side BC is shorter than side CD.	
	*What do we need to remember as we draw the rest of the shape?* I will make sides AB and AD longer than side BC.	
	*Does the shape match the properties in the question?* I will check that my vertex labels are all in the correct order and that the shape has the required properties.	

As well as labelling polygons in this way, mathematicians often refer to 'the general triangle', as shown. Although you use capital letters to refer to vertices, when you are discussing 'the general triangle' you use lower-case letters (not capitals) to label sides. In the diagram, side BC is labelled a because it is opposite angle A.

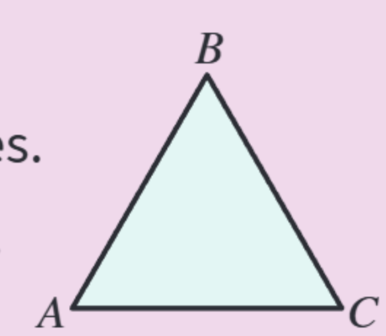


Fluency questions

1 The diagram shows a line segment.

Copy and complete this sentence.
The diagram shows line segment ___.

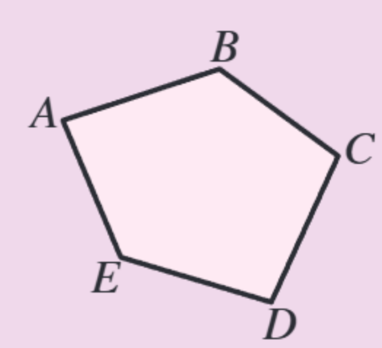
2 The diagram shows a triangle.



Copy and complete these sentences.

a The diagram shows triangle ___.
b Angle ABC is the angle at vertex ___.
c Side ___ is opposite angle BCA.
d Angle ___ is opposite side BC.

3 The diagram shows a polygon.

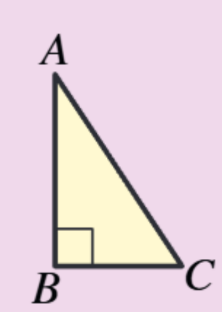


Which of these describes this polygon? Choose all that apply.

a $ABCDE$
b $ACDBE$
c $DEABC$
d $EDCBA$
e $DECBA$
f $BCAED$

Stretch zone

4 The diagram shows a **right-angled triangle**.



Write an expression for the **area** of triangle ABC.

1.1.2 Right-angled triangles

After this topic, you will be able to:

- understand and use terms and notation for right-angled triangles and triangles with equal angles.

Key idea

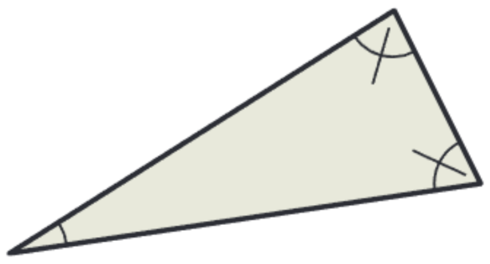
In any right-angled triangle, the **hypotenuse** is the longest side.

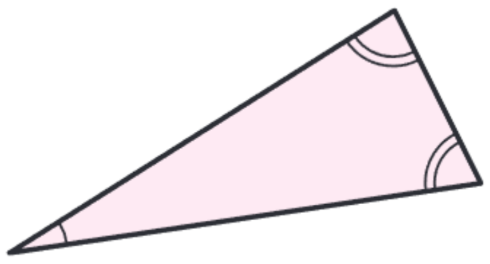
Key words

hypotenuse, mathematical, isosceles triangle, equilateral triangle

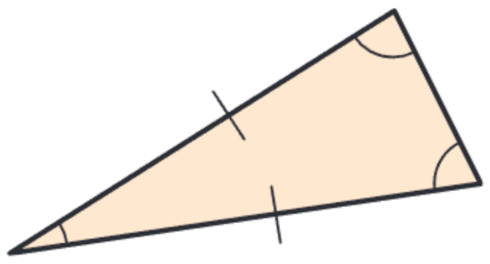
You can use **mathematical** notation (symbols) to show angle relationships in diagrams.

You show that two angles in an **isosceles triangle** are equal by using dashes (short lines) through the arcs (short curves), or by using double arcs.

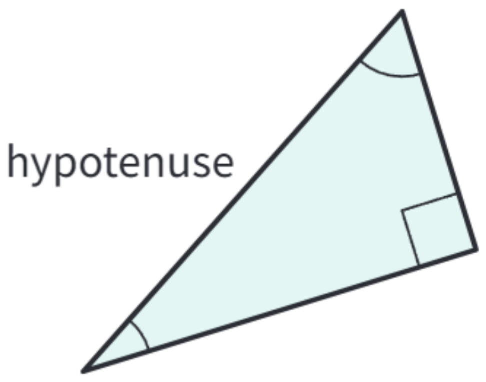




You also use the dash notation to show that two sides are the same length.



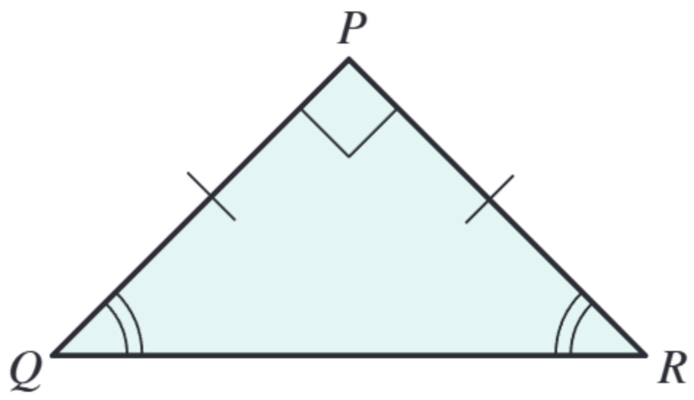
You show a right angle with a small square.



There are relationships between angles and sides within shapes. In right-angled triangles, the side opposite the right angle is called the hypotenuse.

Worked example

The diagram shows triangle PQR.



a **Write down any pairs of sides that are equal in length and any pairs of equal angles.**

b **Does this triangle have a hypotenuse?**

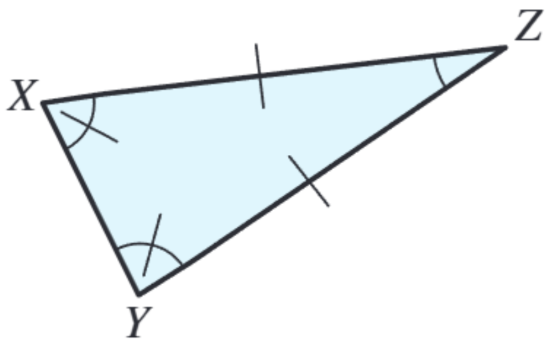
Thinking

What notation shows sides of equal length?

I can check whether any sides have dash marks on them.

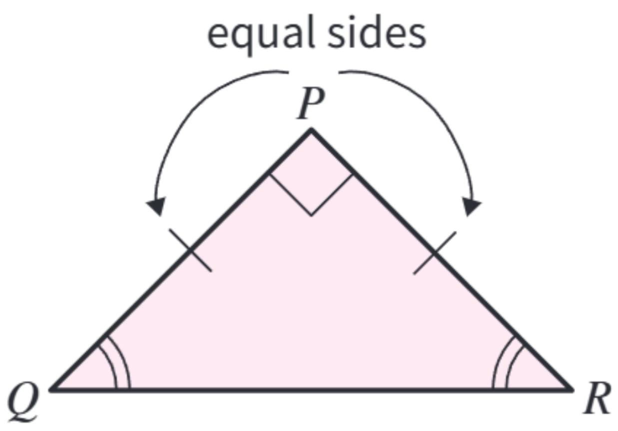
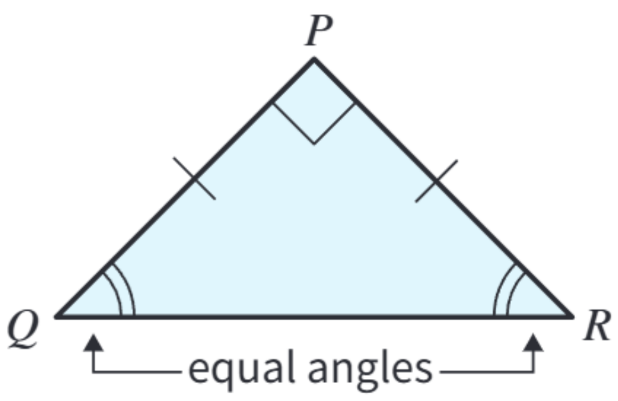
Your turn!

The diagram shows triangle XYZ.



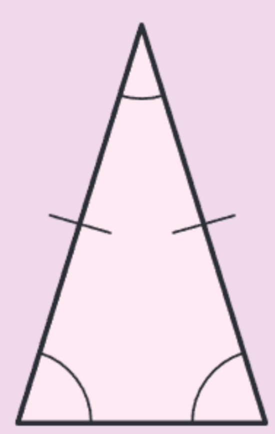
a **Write down any pairs of sides that are equal in length and any pairs of equal angles.**

b **Does this triangle have a hypotenuse?**

Worked example	Thinking	Your turn!
	*What notation shows equal angles?* I can check whether any angles have double arcs or dashes on them.	
	*How do we write the equal sides and equal angles?* I need to use letters for the sides. I need to use the angle symbol and letters for the angles.	
a Sides PQ and PR are equal. Angles $\angle PQR$ and $\angle PRQ$ are equal.		
b QR is the hypotenuse.	*Is PQR a right-angled triangle?* PQR is a right-angled triangle. I know that the side opposite the right angle is the hypotenuse.	

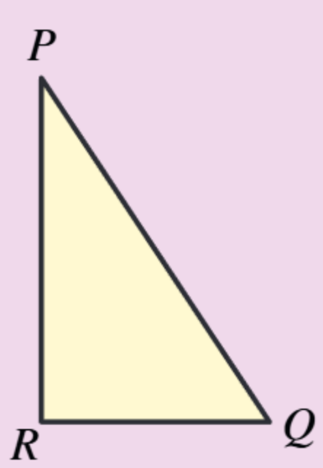
Fluency questions

1 What type of triangle is shown in the diagram?
Explain how you know.

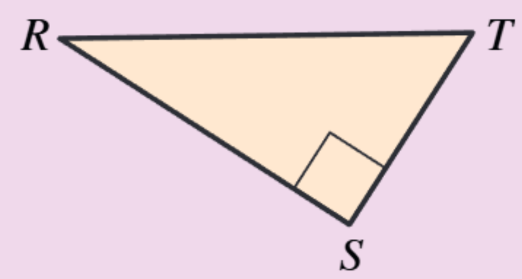


2 Bob and Amrita are both trying to describe how you indicate that a triangle is an **equilateral triangle**.
Bob says, 'You add a dash to each side.'
Amrita says, 'You mark each angle with a double arc.'
Who is correct? Explain your answer.

3 The diagram shows a right-angled triangle.
The hypotenuse of the triangle is the side PQ.
Which angle of the triangle is the right angle?
Explain how you know.



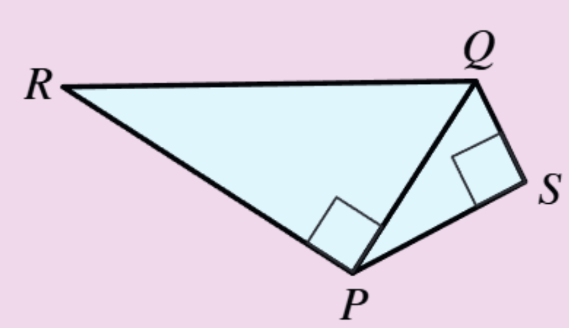
4 Which side of the triangle is the hypotenuse?
Explain how you know.



Stretch zone

5 **a** Choose the correct word to complete each of these sentences about a right-angled triangle.
 i The right angle is the largest/smallest/middle angle.
 ii The two angles that are not right angles are acute/obtuse/reflex angles.
b Explain your answers to part **a**.

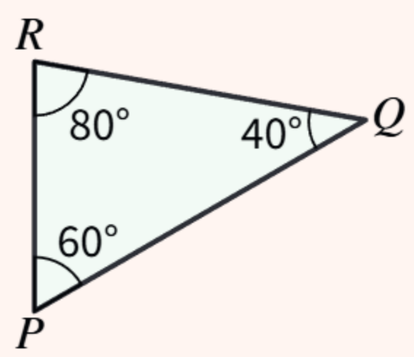
6 Which triangle has PQ as the hypotenuse?
Explain how you know.



1.1 Intelligent practice

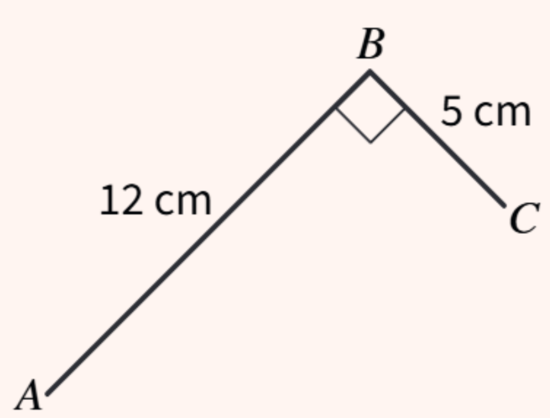
In each question, you might notice something when you move from one question part to the next. What is different between each question part (e.g. **1b**) and the one that came before (e.g. **1a**)? Decide how you expect the answer to be different. Then work through the question and check your answer. Think about why your prediction was right or wrong.

1 Is each statement true or false? Use the diagram.

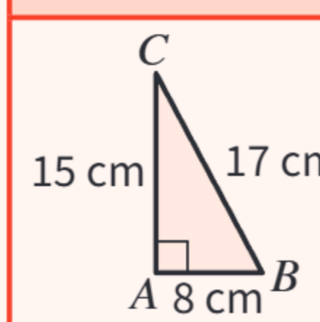
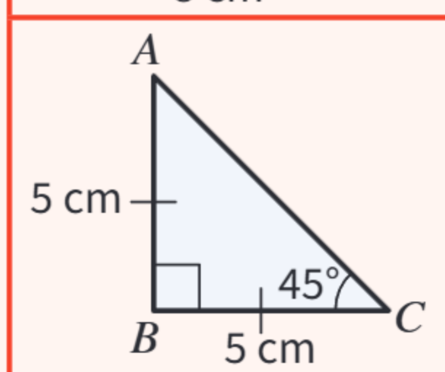
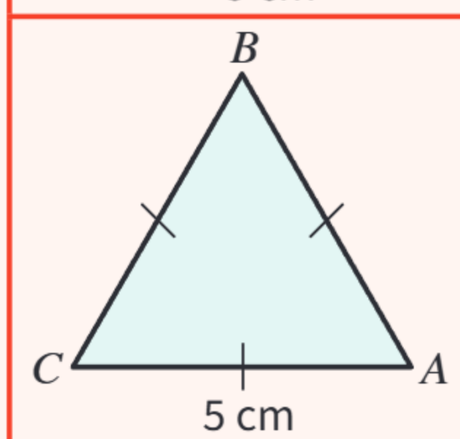


a Angle RPQ is 60°.
b Angle QPR is 60°.
c $\angle QPR = 60°$.
d Angle PQP is 40°.
e Angle PQR is 40°.
f PR is longer than PQ.
g $R\hat{P}Q$ is bigger than RQ.

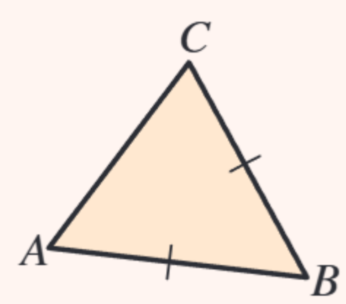
2 Use the diagram to answer the following questions.
a How long is AB?
b What is the size of angle ABC?
c Draw the diagram and measure CA. Use a ruler and protractor.

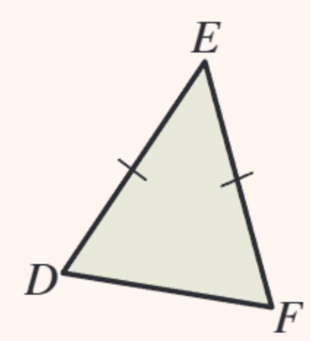


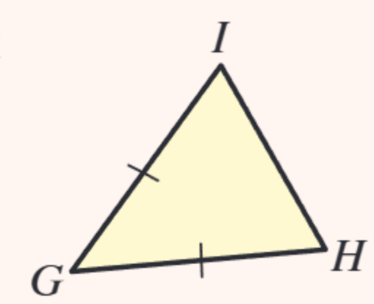
3 Copy and complete the table for each triangle.

Diagram	$AB =$	$CB =$	$B\hat{A}C =$
			
			
			

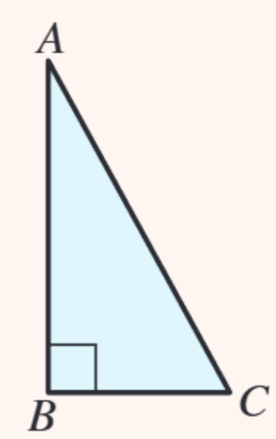
4 For each isosceles triangle, write down the pair of equal angles.

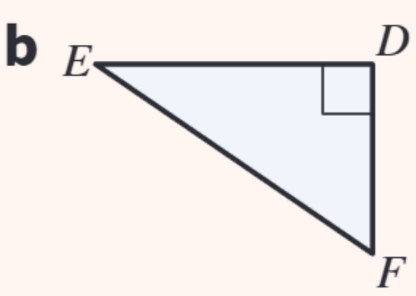
a


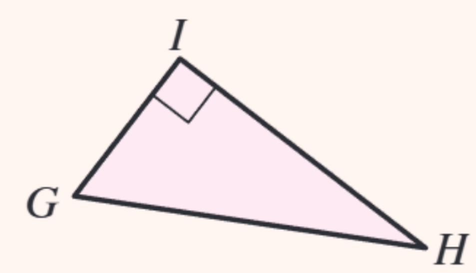
b


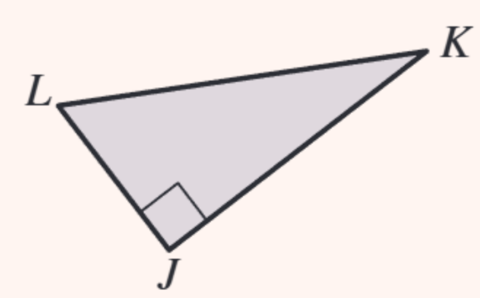
c


5 For each right-angled triangle, write down which side is the hypotenuse.

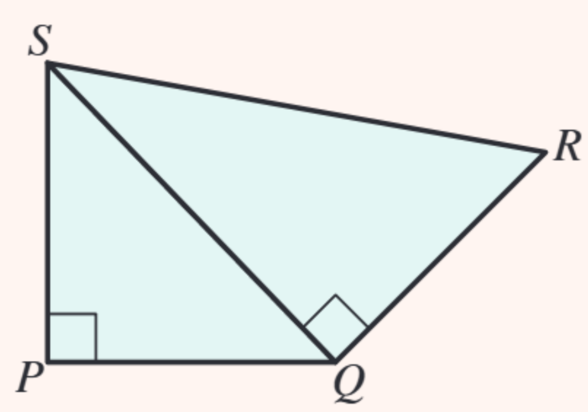
a


b


c


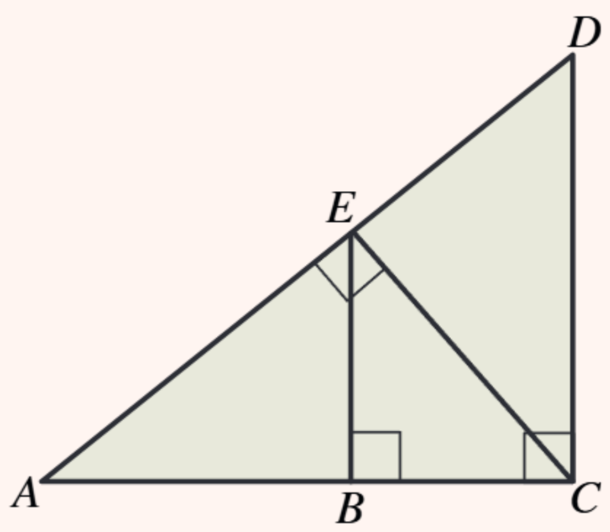
d


6 $PQRS$ is made of two right-angled triangles.



a What is the hypotenuse of triangle PQS?
b What is the hypotenuse of triangle RQS?
c Why does triangle PRS not have a hypotenuse?

7 In the diagram below, AED is a straight line. Use the diagram to answer the questions.

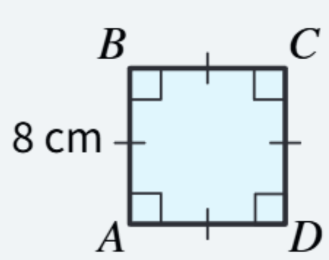


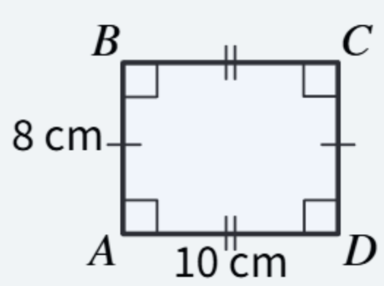
a What is the hypotenuse of triangle ABE?
b What is the hypotenuse of triangle ACE?
c What is the hypotenuse of triangle EDC?
d What is the hypotenuse of triangle ADC?
e On a copy of the diagram sketch point F, so that ED is the hypotenuse of triangle DEF.

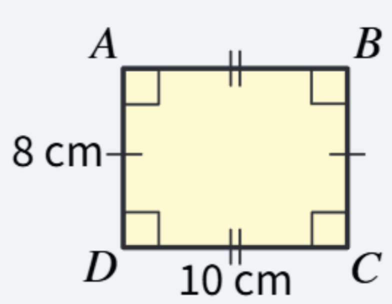
1.1 Which method?

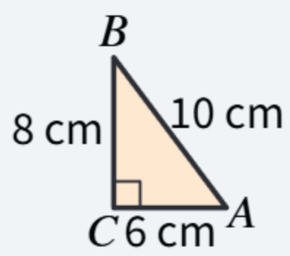
In these questions, you will need to think carefully about which methods to apply.
For some questions, you might need to use skills from Student Book 7 or Student Book 8.

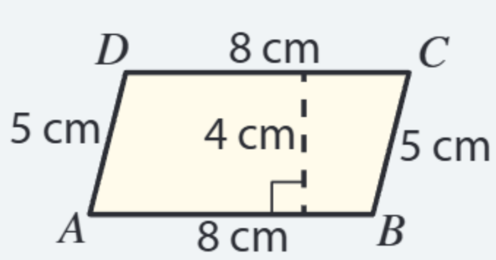
1 Copy and complete the table for each shape. Part **d** has been done for you.

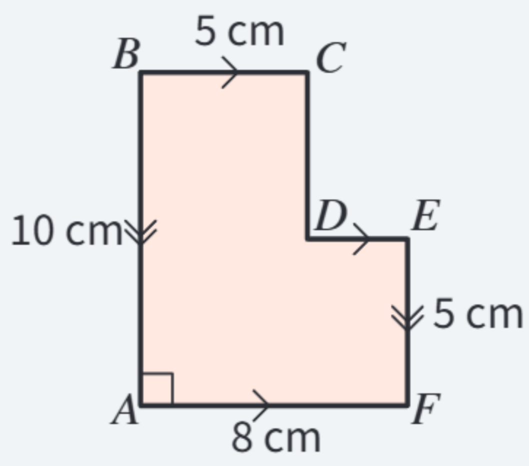
a


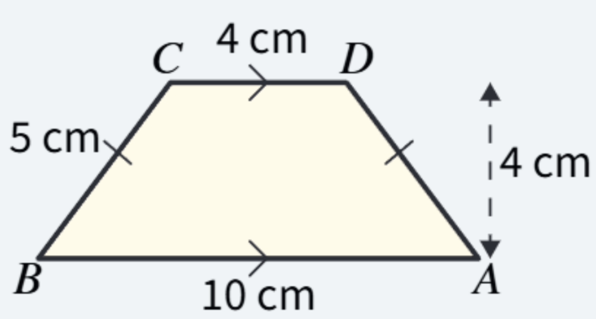
b


c


d


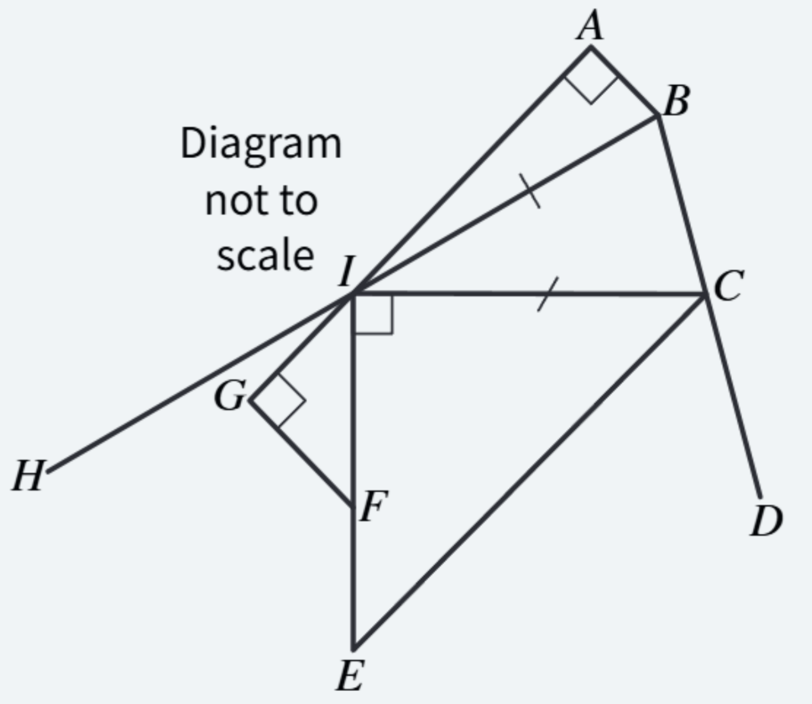
e


f


g


Shape	Type of polygon	Length of AB	Length of BC	Type of angle $A\hat{B}C$	Perimeter	Area
a						
b						
c						
d	Right-angled triangle	10 cm	8 cm	Acute	24 cm	24 cm^2
e						
f						
g						

2 In the diagram, $\angle DCE = 65°$, $\angle EFG = 120°$, $\angle HIA = 150°$, and $IB = IC$.



Using facts about angles in triangles and angles on a straight line, find the value of:

a $\angle GFI$

b $\angle GIF$

c $\angle AIB$

d $\angle ABI$

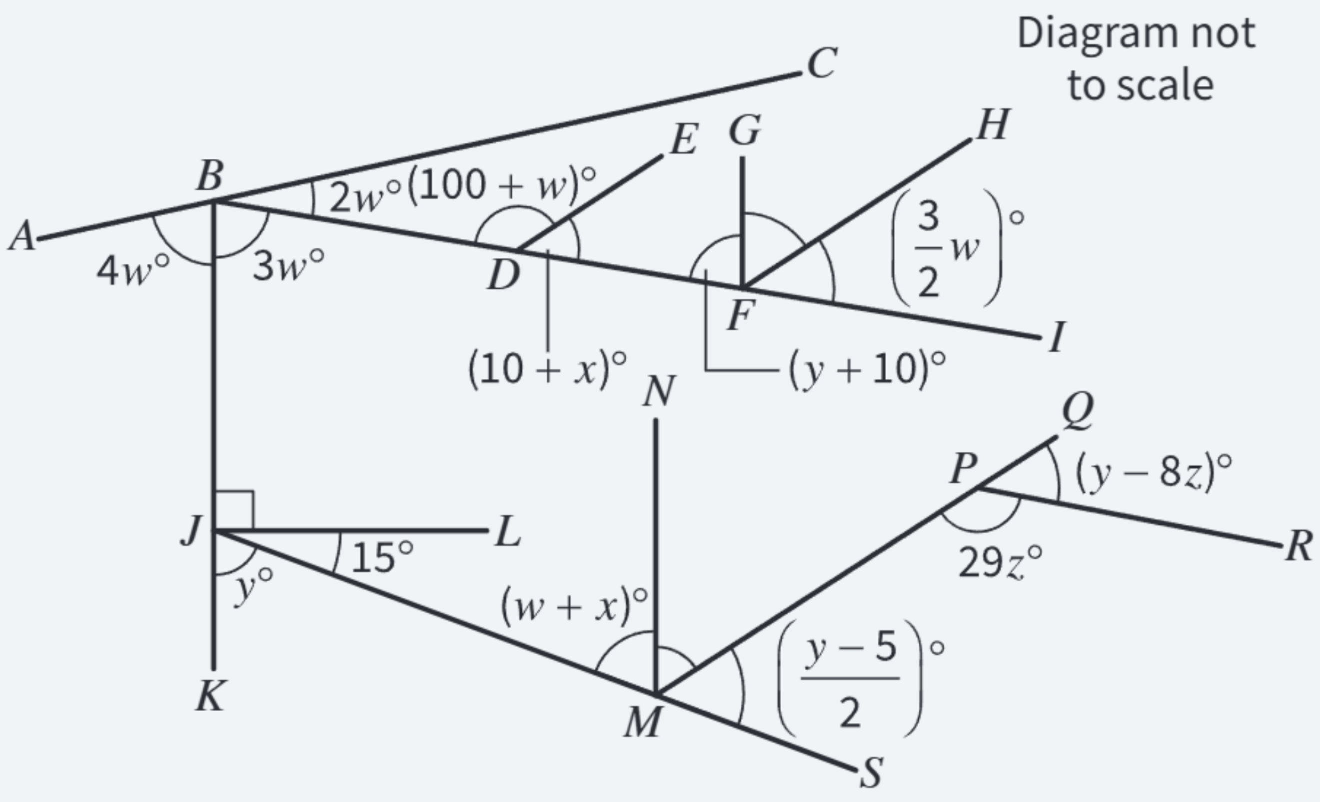
e $\angle CIB$

f $\angle BCI$

g $\angle ECI$

h $\angle CEI$.

3 Look at the diagram.



a Find the value of each unknown.

- **i** w
- **ii** x
- **iii** y
- **iv** z

b Find the value of each angle.

- **i** $G\hat{F}H$
- **ii** $\angle GFI$
- **iii** $\angle NMP$
- **iv** $N\hat{M}Q$

c Which two of the line segments are parallel?

1.1 Expert practice

There might be more than one way to look at these questions.
Once you have answered a question one way, can you think of another way?

1 Shade in each of the following polygons on a copy of the diagram.

a Triangles:

i *ZUP*

ii *KBW*

iii *DYL*

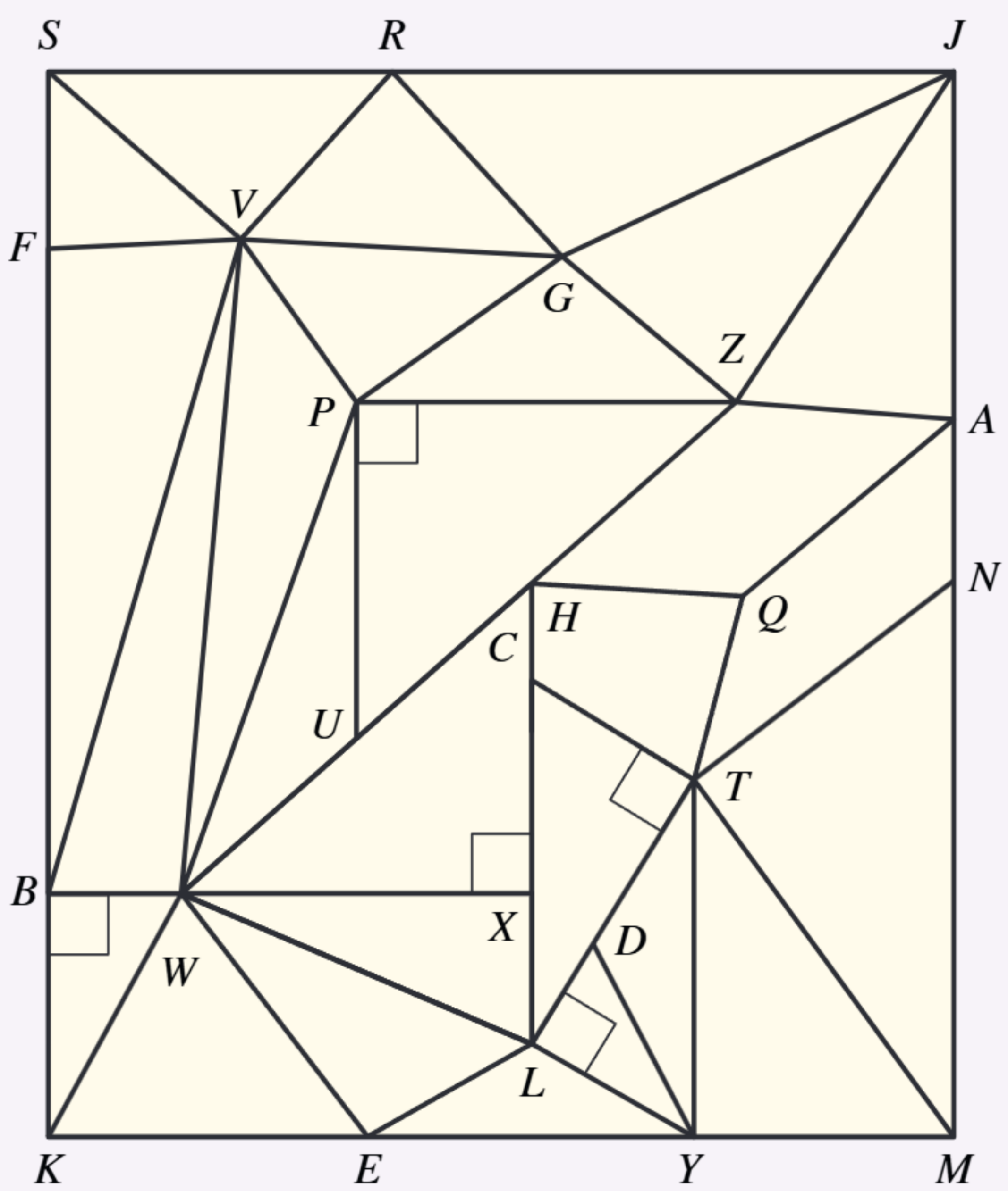
iv *LTC*

v *WXH*

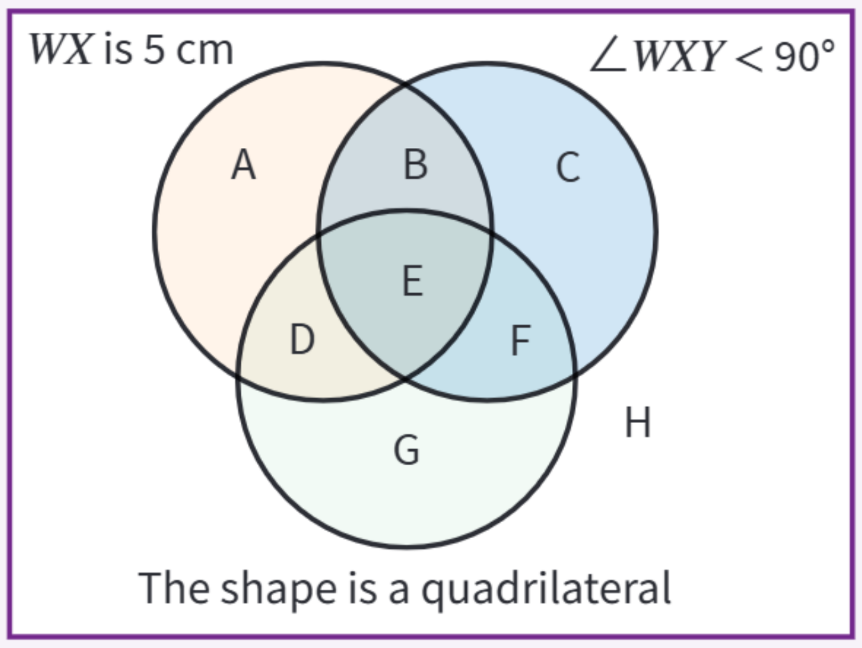
b Quadrilaterals:

i *ZAQH*

ii *PGRV*



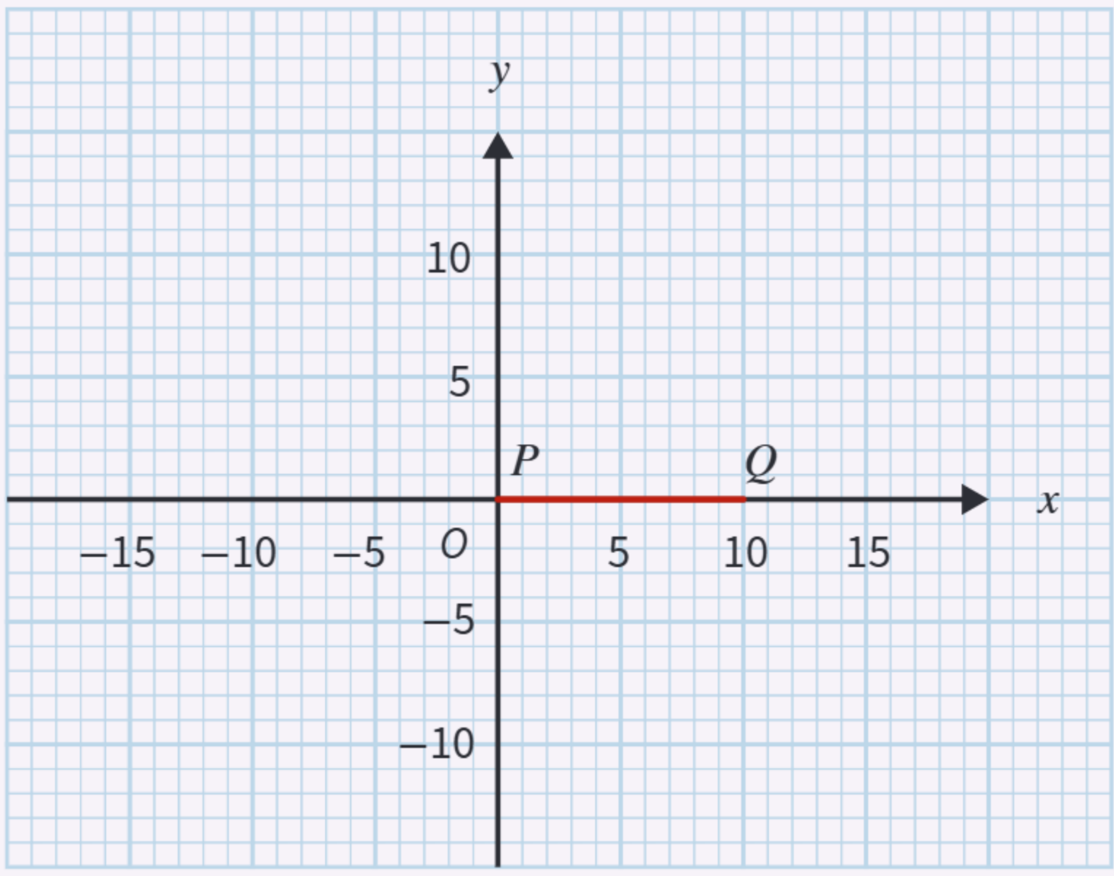
2 Draw and label a polygon that satisfies each section of the Venn diagram.



3 The coordinate grid shows line segment PQ. For each condition below:

i write a pair of coordinates that satisfy the condition

ii sketch a diagram to show all of the coordinate points that satisfy the condition.



a A is a point where triangle APQ is a right-angled triangle.

b B is a point where triangle BPQ is an isosceles triangle.

c C is a point where line segment PC makes a 45° angle with line segment PQ.

d D is a point where line segment DQ makes a 45° angle with line segment PQ.

e E is a point where line segment PE is shorter than line segment PQ.

f F is a point where line segment PF is longer than line segment PQ.

1.2 Similarity

1.2.1 Similar shapes

After this topic, you will be able to:
- identify similar shapes.

Key idea

Two shapes are **similar** if their respective (matching) angles are the same and their respective sides are in the same proportion. Each shape is an **enlargement** of the other.

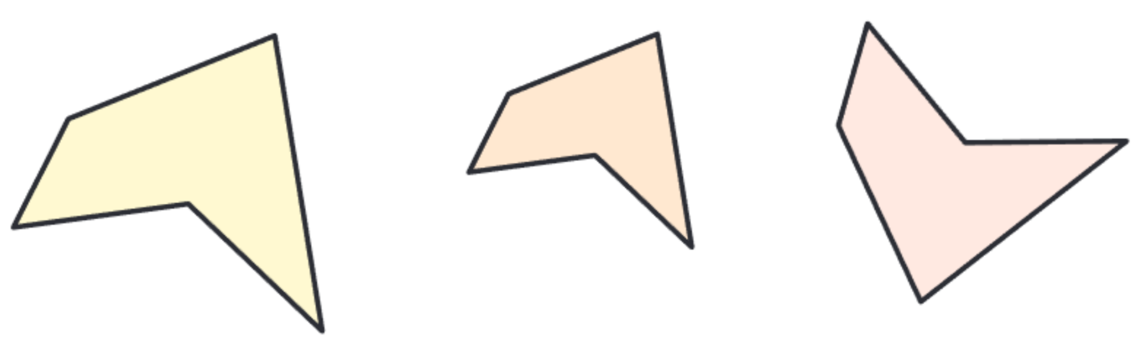
Key words

similar, enlargement, similarity, radius (plural: radii), transformation, rectangle, cuboid, cube, sphere

Notation and labelling can show how particular properties of shapes are related to each other.

One important relationship between shapes is **similarity**.

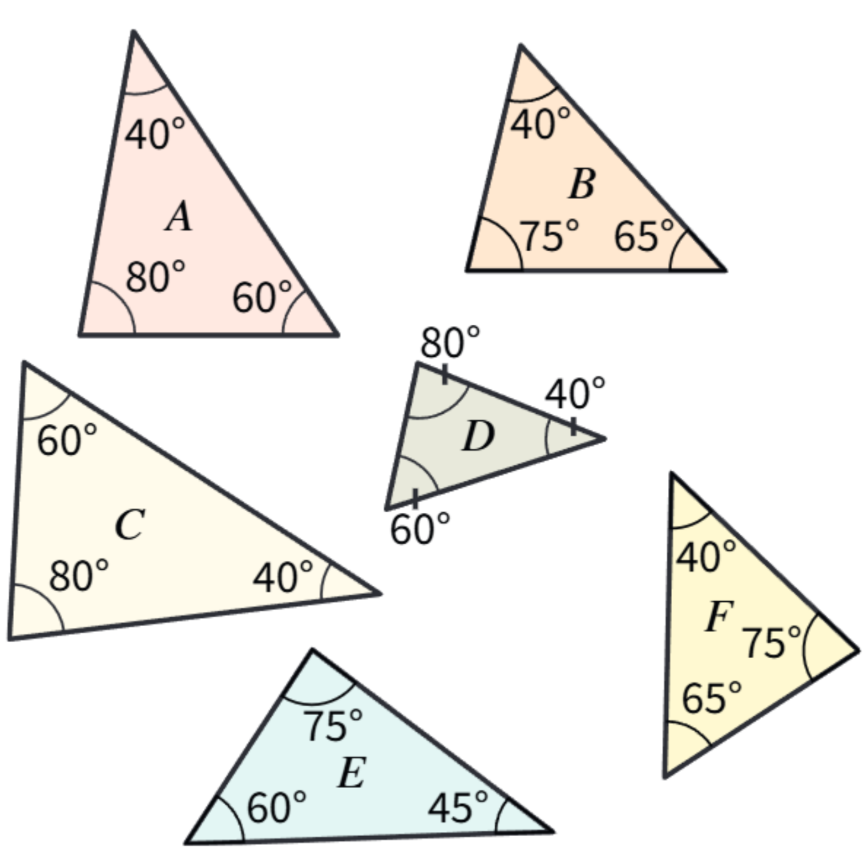
Shapes can be similar even if they are rotated (turned) or reflected. These shapes are similar. The proportions of the sides and the angles are the same.



All circles are similar. This is because the **radii** are always in proportion with one another.

Worked example

Identify which triangles below are similar.



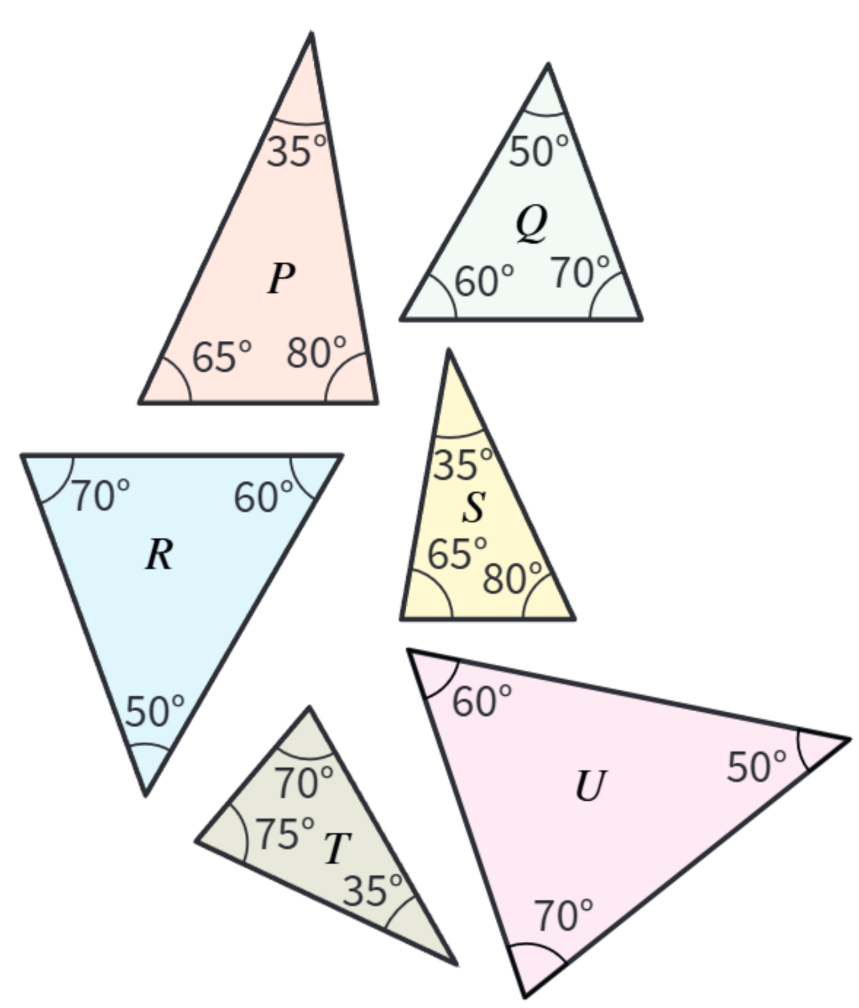
Thinking

How do we identify similar triangles?

I need to identify the triangles that have all the same angles.

Your turn!

Identify which triangles below are similar.



Worked example	Thinking	Your turn!
The angles in A, C, and D are 40°, 60°, and 80°. The angles in B and F are 40°, 65°, and 75°.	*Which triangles are similar?* I know that if the triangles have identical angles, then those triangles are similar.	
Triangles A, C, and D are similar. Triangles B and F are similar.		

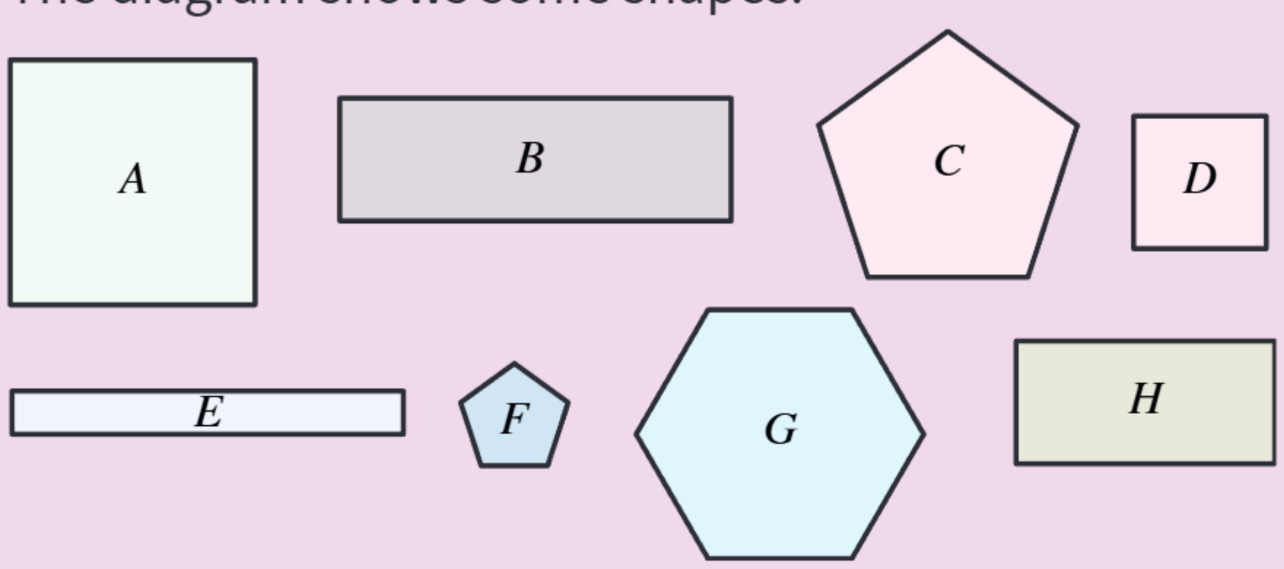
Any enlargement of an object will give you an image that is a similar shape to the original object. See Student Book 7, Chapter 9 for more about **transformations**.

Fluency questions

1 Decide if each statement is true or false.

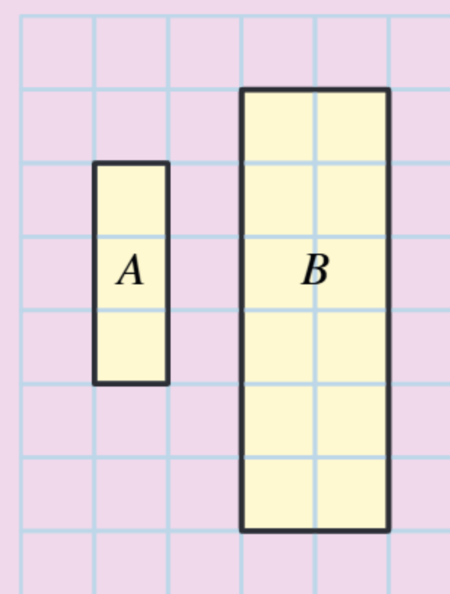
a Similar shapes have the same angle sizes.
b Similar shapes have the same side lengths.
c Similar shapes have sides in proportion to each other.
d Similar shapes can have different angle sizes.

2 The diagram shows some shapes.

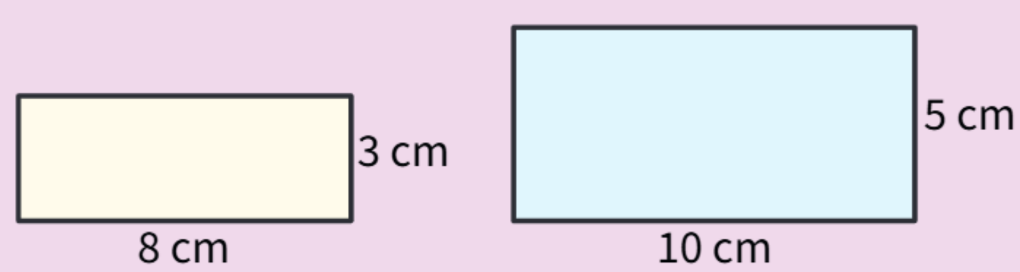


Write down any pairs of shapes that are similar.

3 Shapes A and B are similar. Describe how you could transform shape A to shape B.



4 The diagram shows two **rectangles**.



Are the rectangles similar? Explain your answer.

Stretch zone

5 Decide if each statement is true or false. Explain your answer in each case.

a All circles are similar.
b All **cuboids** are similar.
c All **cubes** are similar.
d All **spheres** are similar.

1.2.2 Angles in similar shapes

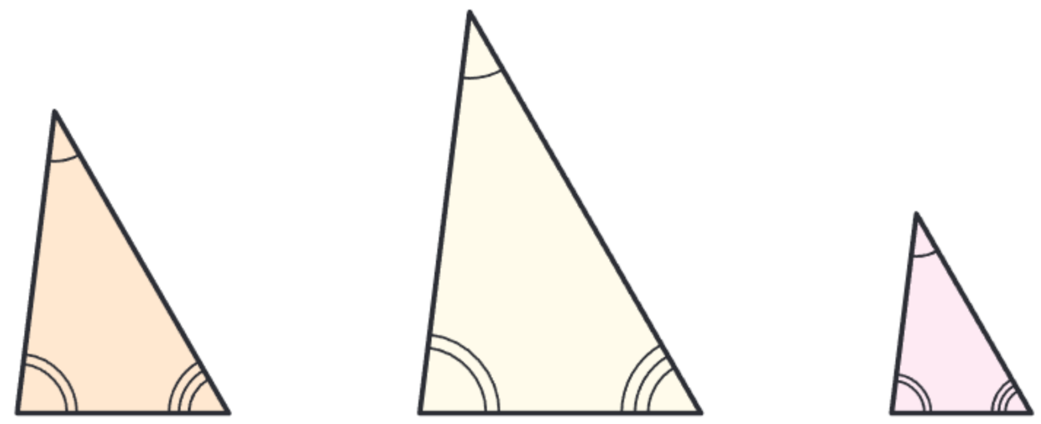
After this topic, you will be able to:

- find missing angles in similar shapes.

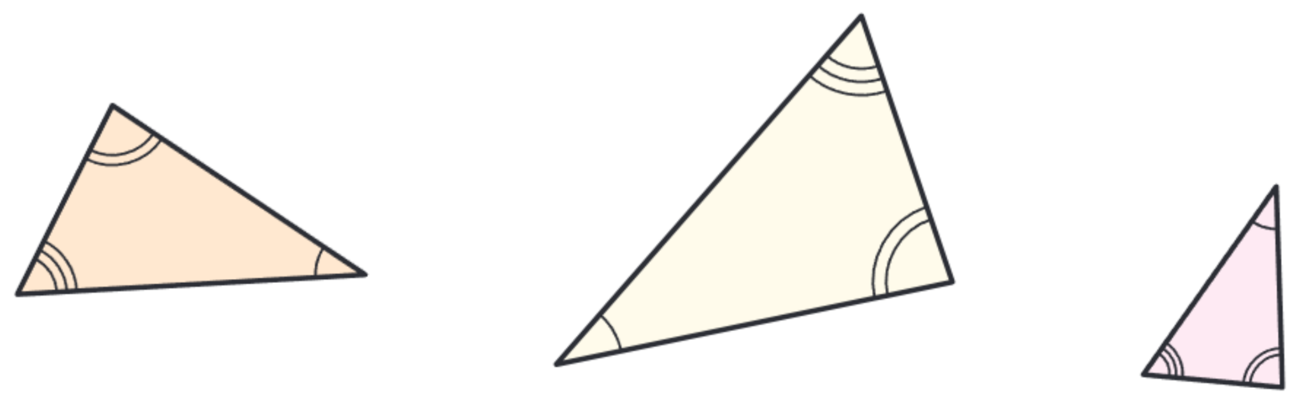
Key idea

You can use the fact that angles in similar shapes are equal to find missing angles in some shapes.

If two shapes are similar, their respective (matching) angles will be the same. In all the similar triangles below, the equal angles are shown by matching arcs.



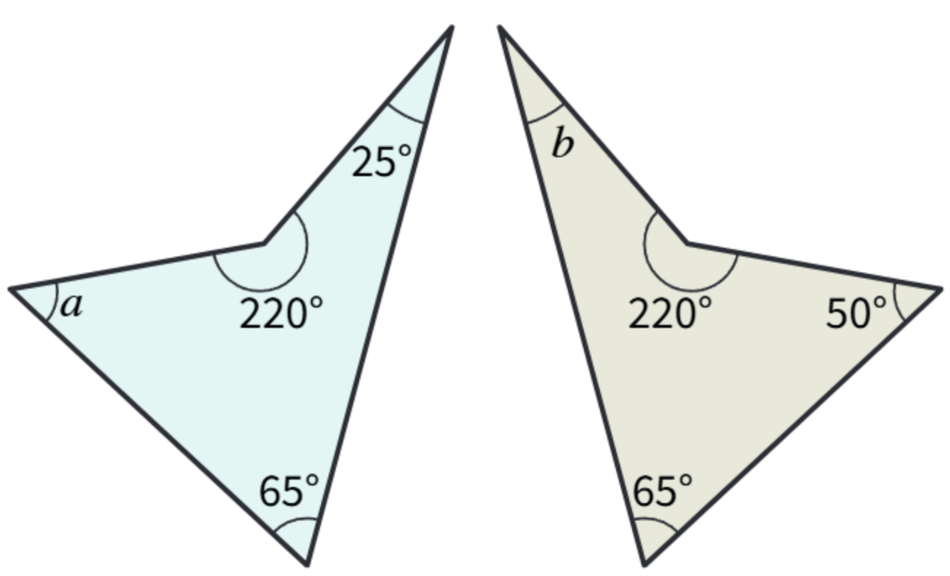
This is still true when similar shapes are rotated or reflected.



You could work out missing angles in a triangle using angle sums, but it is more efficient to use what you know about similarity.

Worked example

The two polygons are similar.
Find the sizes of angles a and b.



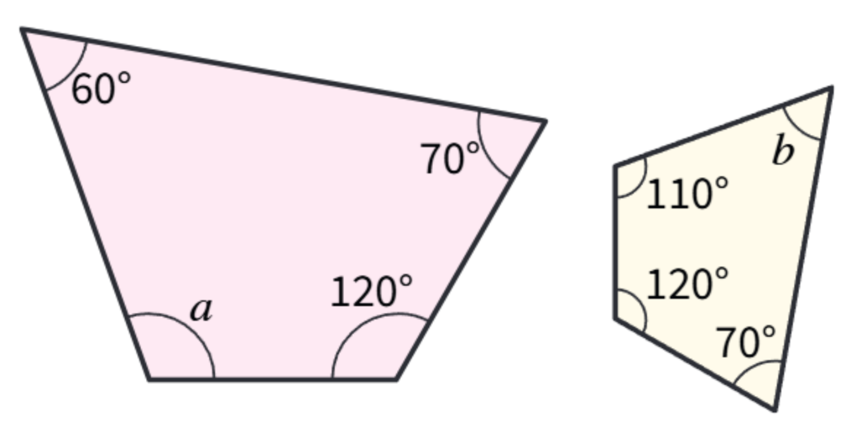
Thinking

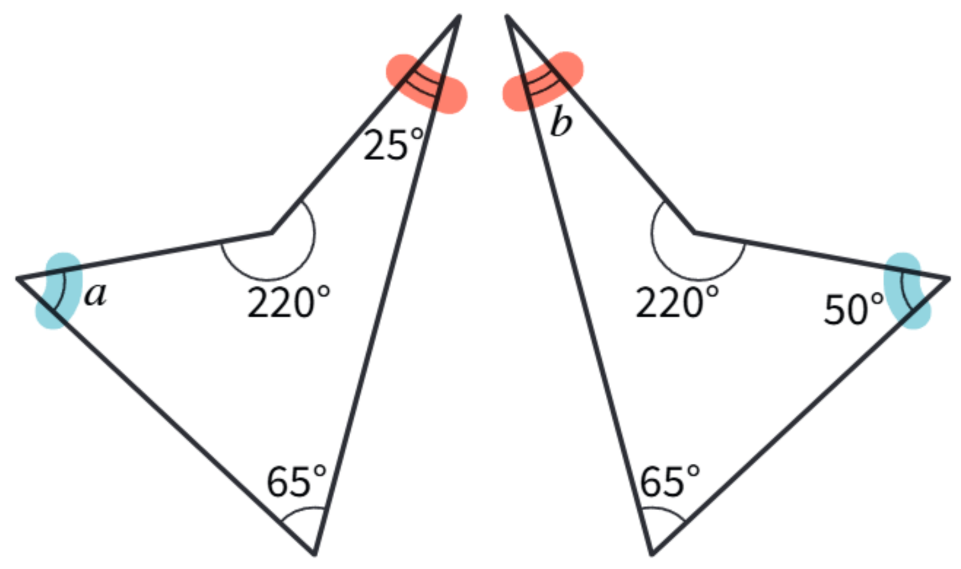
What do we know about the polygons?

I know that the polygons are similar and so the angles in both polygons must be the same.
I can identify the angles that are equal.

Your turn!

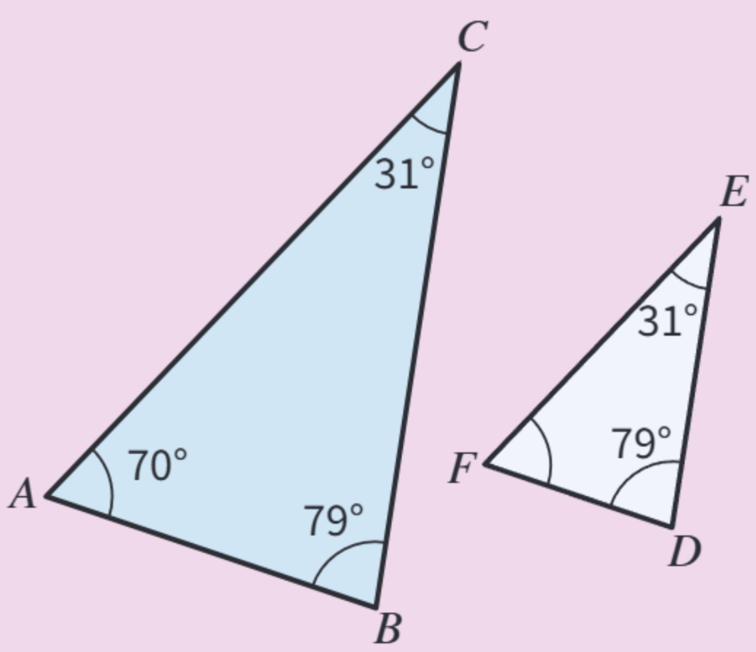
The two polygons are similar.
Find the sizes of angles a and b.



Worked example	Thinking	Your turn!
	*Which angles are related in the two polygons?* I can match the angles that are equal, so I can see that $a = 50°$ and $b = 25°$.	
$a = 50°$ and $b = 25°$.		

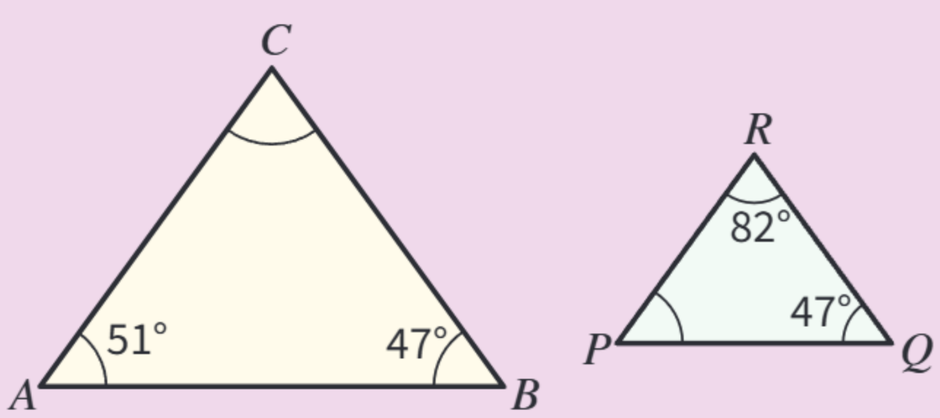
Fluency questions

1 These two triangles are similar.



Find the size of angle EFD.

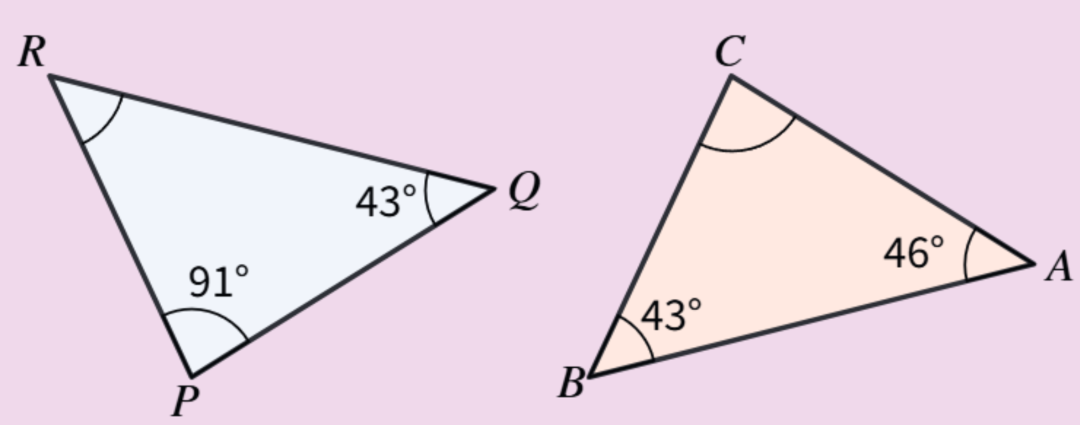
2 These two triangles are similar.



Find the size of:

a angle ACB **b** angle RPQ.

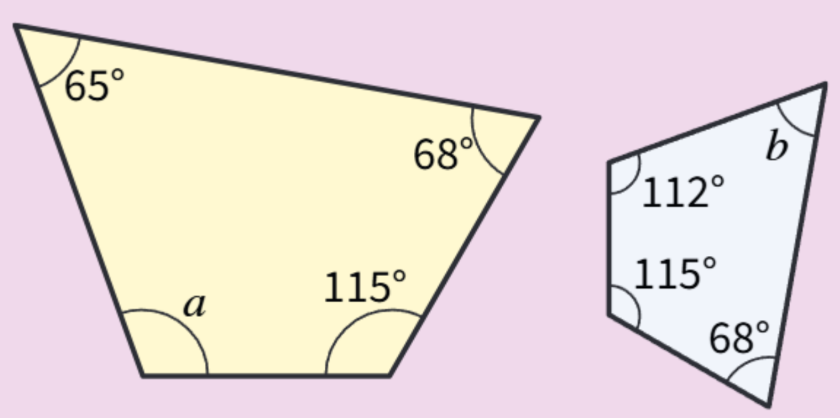
3 These two triangles are similar.



Find the size of:

a angle PRQ **b** angle ACB.

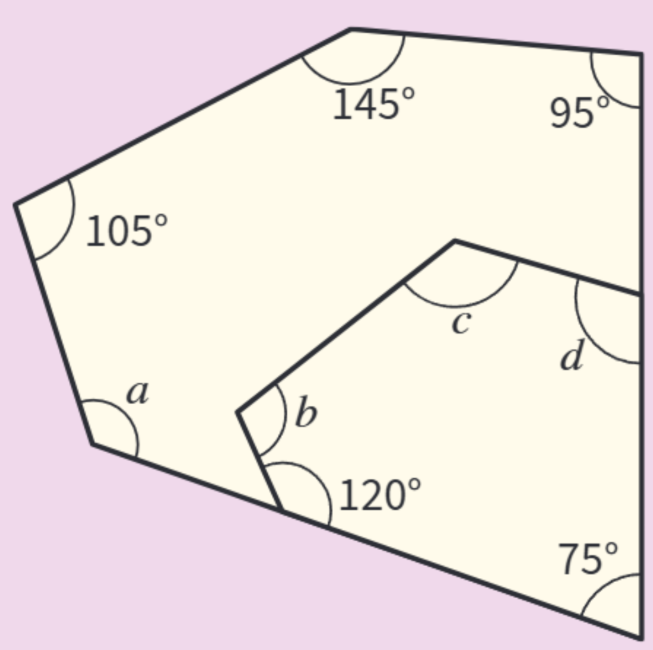
4 These two quadrilaterals are similar.



Find the size of angles a and b.

Stretch zone

5 These two similar shapes share a vertex.



Find the size of angles a, b, c, and d.

1.2.3 Side lengths in similar shapes

After this topic, you will be able to:

- find missing lengths in similar shapes.

Key idea

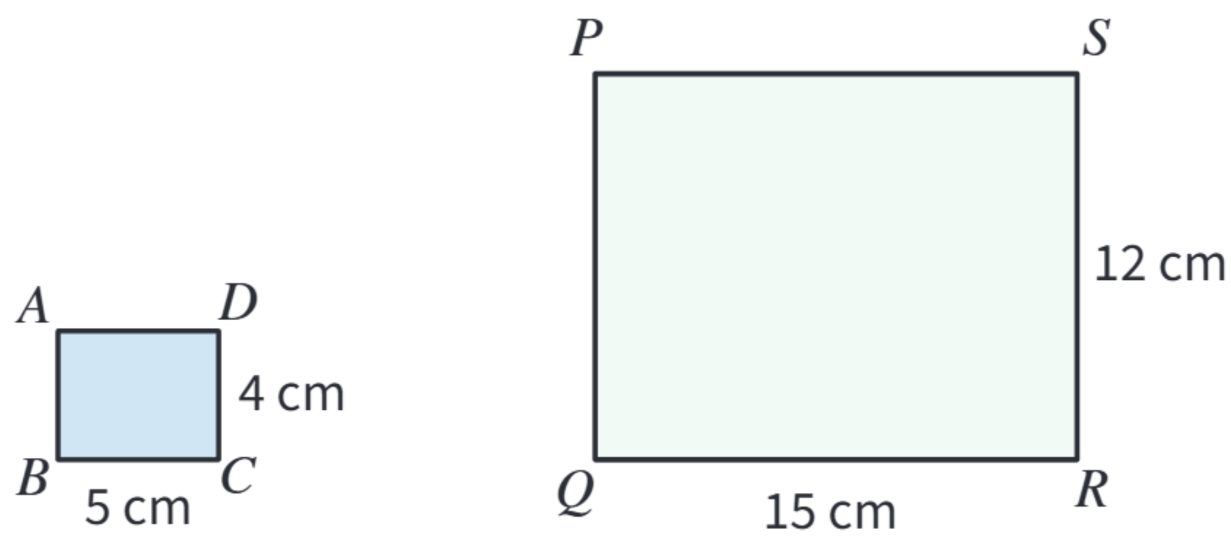
In similar shapes, the **multiplicative relationship** is the same between each pair of matching sides.

Key words

multiplicative relationship, scale factor, division, multiplication, integer, unit fraction, reciprocal, trapezium

Two shapes are similar if their respective (matching) angles are the same and their respective sides are in the same proportion.

These two rectangles are similar.



By looking at one pair of related sides, you can see that one side is 3 times longer than the other.

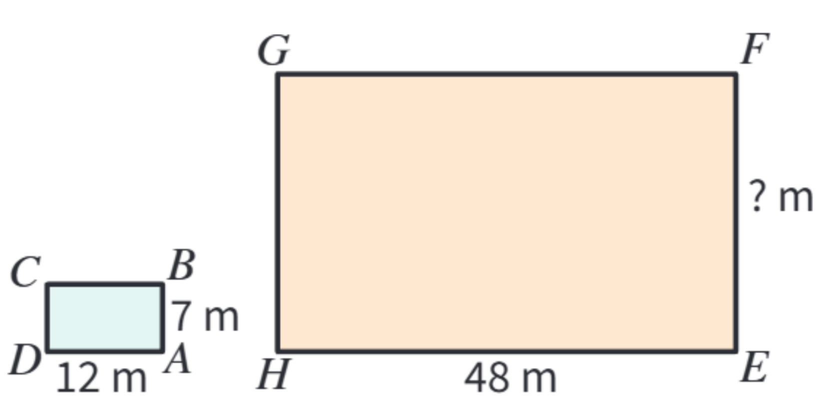
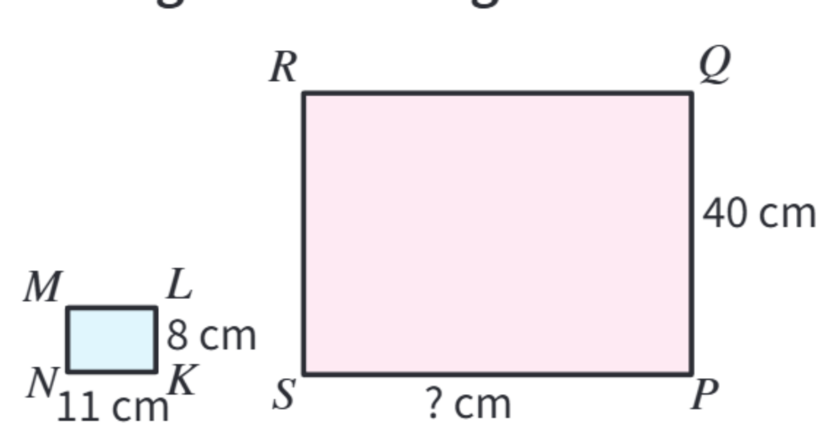
For example, BC is 5 cm and QR is 15 cm:

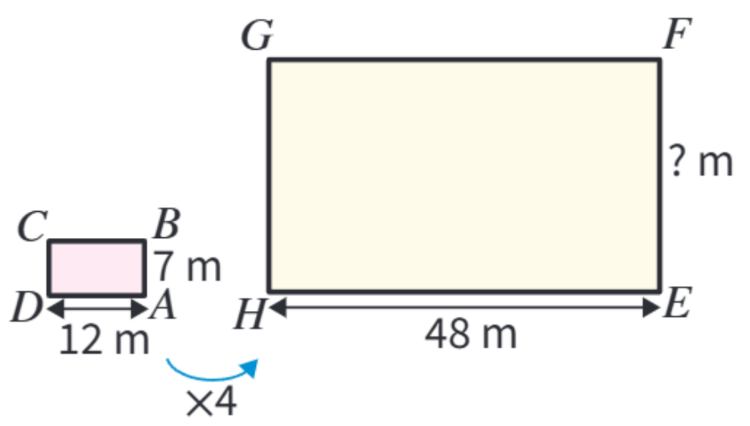
$5 \times 3 = 15$

This relationship is the **scale factor** of the enlargement of rectangle $ABCD$ to $PQRS$. It is true for all the matching sides in the shapes.

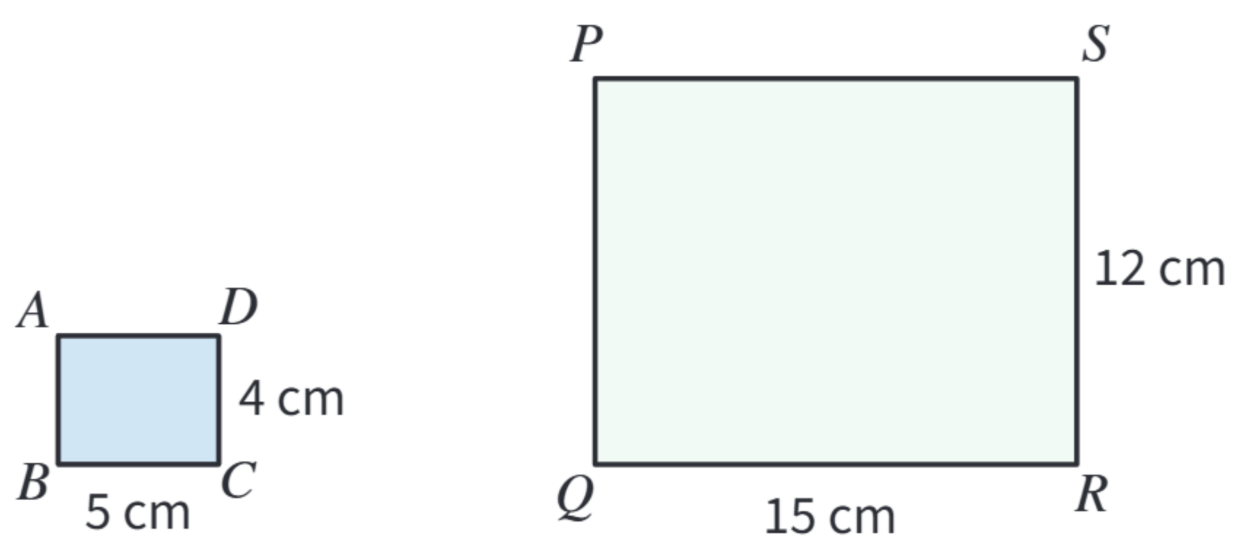
$AB \times 3 = PQ$ $CD \times 3 = RS$

$BC \times 3 = QR$ $AD \times 3 = PS$

Worked example	Thinking	Your turn!
$ABCD$ and $EFGH$ are similar rectangles. Find length EF. 	*What do we know about the rectangles?* I know that the rectangles are similar, and so I can identify how the sides in $ABCD$ relate to the sides in $EFGH$.	**$KLMN$ and $PQRS$ are similar rectangles. Find length PS.** 

<table>
<tr><th>Worked example</th><th>Thinking</th><th>Your turn!</th></tr>
<tr><td>AD relates to EH.
AB relates to EF.
BC relates to FG.
CD relates to GH.</td><td>How can we work out the scale factor that enlarges ABCD to EFGH?
I know length AD and I know length EH. I can see that
$4 \times$ length $AD =$ length EH
$4 \times 12 = 48$
The scale factor is 4.</td><td></td></tr>
<tr><td>
</td><td>How do we use the scale factor to find the missing length?
To work out length EF, I need to multiply length AB by the scale factor:
$7 \times 4 = 28$</td><td></td></tr>
<tr><td>$7 \times 4 = 28$ m
EF is 28 m long.</td><td></td><td></td></tr>
</table>

You can use a scale factor to describe the change from the smaller rectangle to the larger rectangle. The scale factor from rectangle $ABCD$ to rectangle $PQRS$ is 3.



You can also find a scale factor for the change from rectangle $PQRS$ to rectangle $ABCD$.

Here is the relationship between the sides:

$AB = PQ \div 3$ $\qquad$ $AD = PS \div 3$

The length of a side of $PQRS$ is **divided** by 3 to give the length of the matching side of $ABCD$.

Scale factors are always shown as **multiplications**.

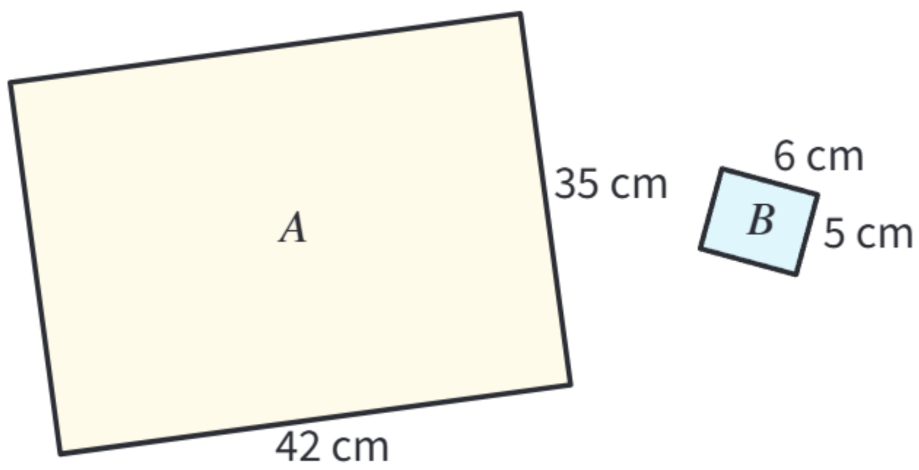
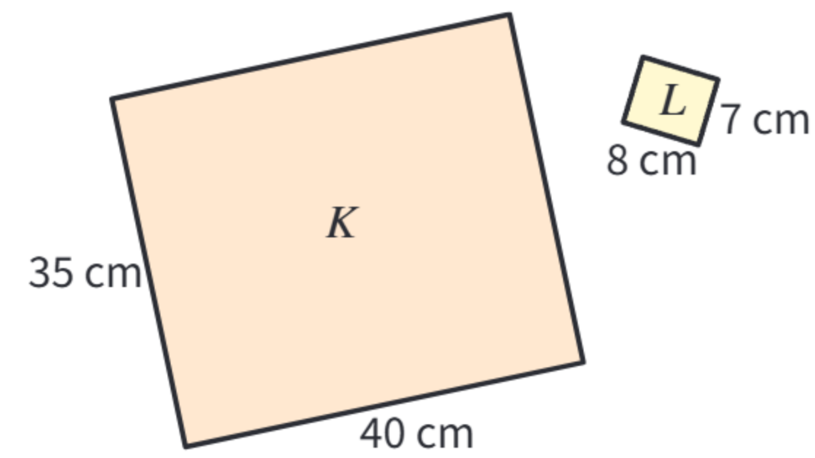
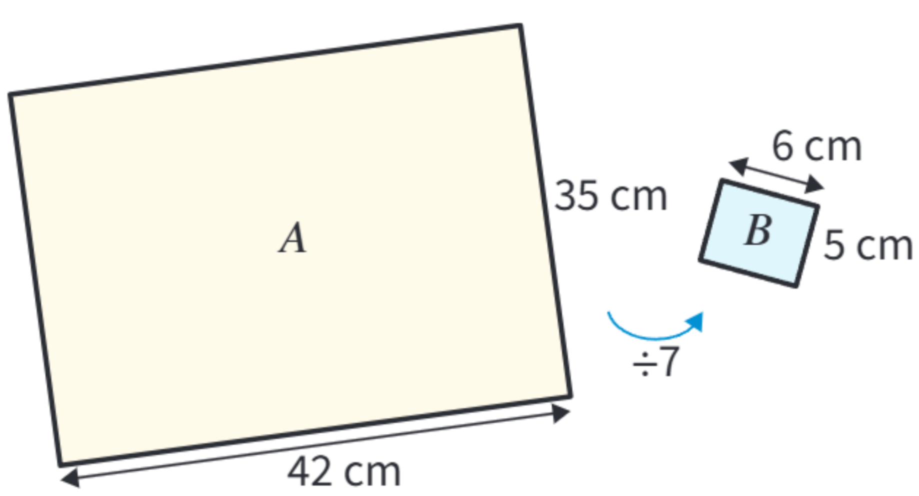
Dividing by 3 has the same effect as multiplying by $\frac{1}{3}$ so:

$AB = PQ \div 3$ $\qquad$ $AD = PS \div 3$

$= \frac{1}{3} \times PQ$ $\qquad$ $= \frac{1}{3} \times PS$

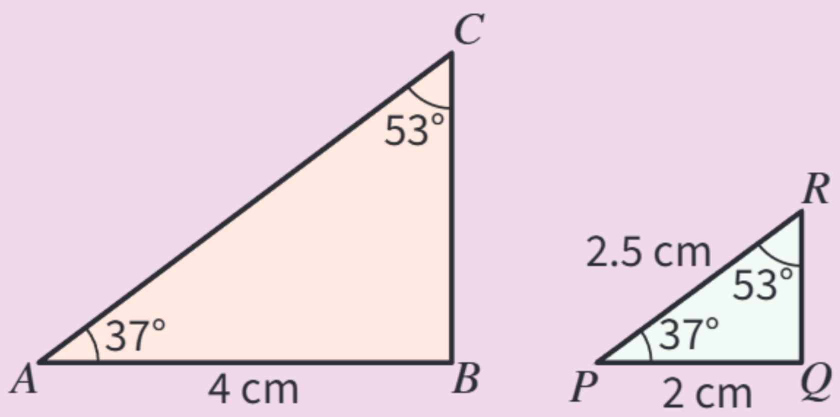
The scale factor for reducing rectangle $PQRS$ to rectangle $ABCD$ is $\frac{1}{3}$.

If you have an **integer** scale factor for the relationship between shape A and shape B, then the scale factor for the relationship between shape B and shape A will always be a **unit fraction**.

Worked example	Thinking	Your turn!
A and B are similar rectangles. Find the scale factor from A to B. 	*What do we know about rectangles A and B?* I know that A and B are similar and that A is larger than B. I can identify how the sides in A relate to the sides in B.	**K and L are similar rectangles. Find the scale factor from K to L.** 
The 42 cm side in A relates to the 6 cm side in B. The 35 cm side in A relates to the 5 cm side in B.	*What is the relationship between the lengths of the sides in A and B?* I know that $42 \div 6 = 7$ and that $35 \div 5 = 7$, so the change from A to B is to divide by 7.	
	*How can we work out the multiplicative relationship from A to B?* I need to find the multiplication that has the same effect as dividing by 7. I can use what I know about **reciprocals**.	
Dividing by 7 has the same effect as multiplying by $\frac{1}{7}$.	*How do we write the scale factor that describes the change from A to B?* Multiplying the side lengths in A by $\frac{1}{7}$ will give the side lengths in B.	
The scale factor from A to B is $\frac{1}{7}$.		

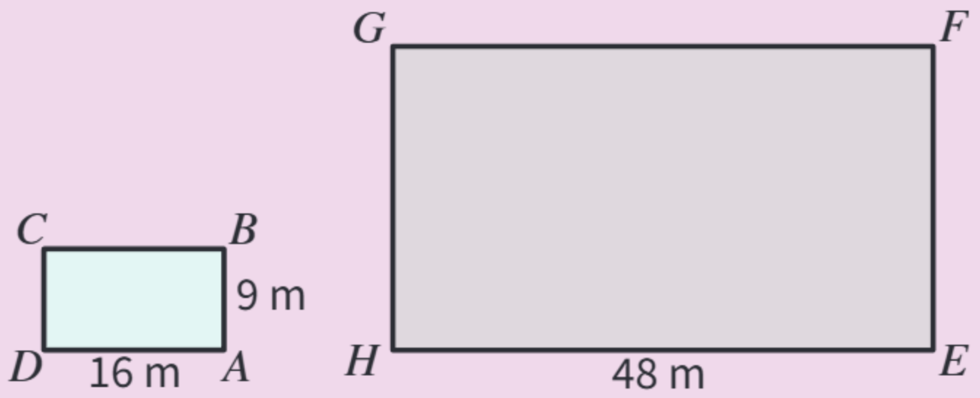
Fluency questions

1 These two triangles are similar.



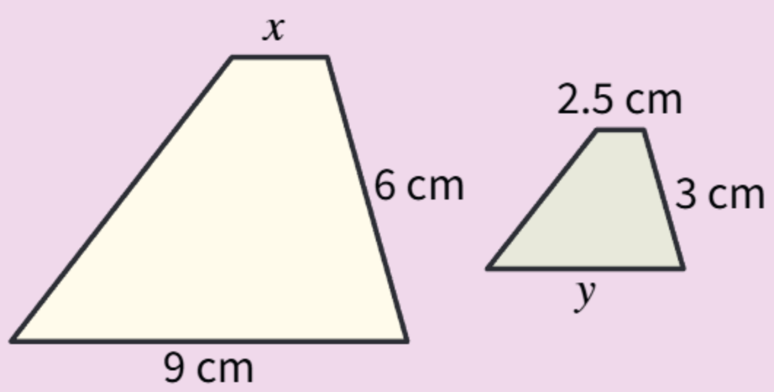
Find the length of side AC.

2 These two rectangles are similar.



Find the length of side EF.

3 These two **trapezia** are similar.

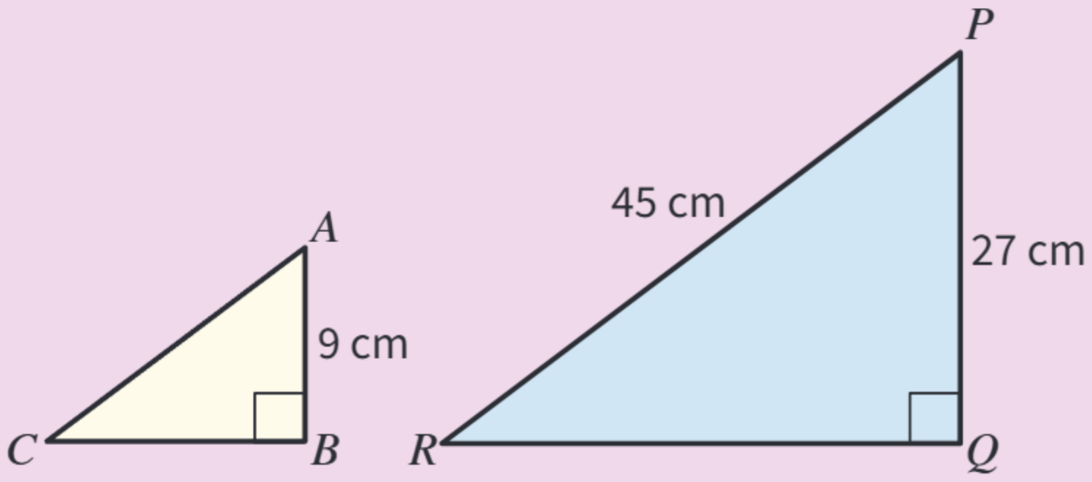


Find the value of:

a x

b y.

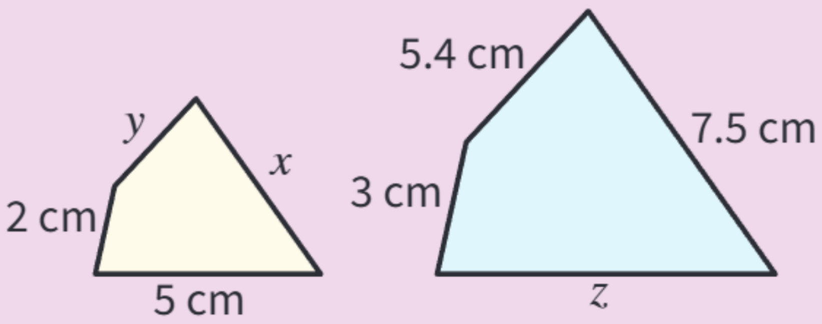
4 These two triangles are similar.



Find the length of side AC.

Stretch zone

5 These two quadrilaterals are similar.



Find the value of:

a x

b y

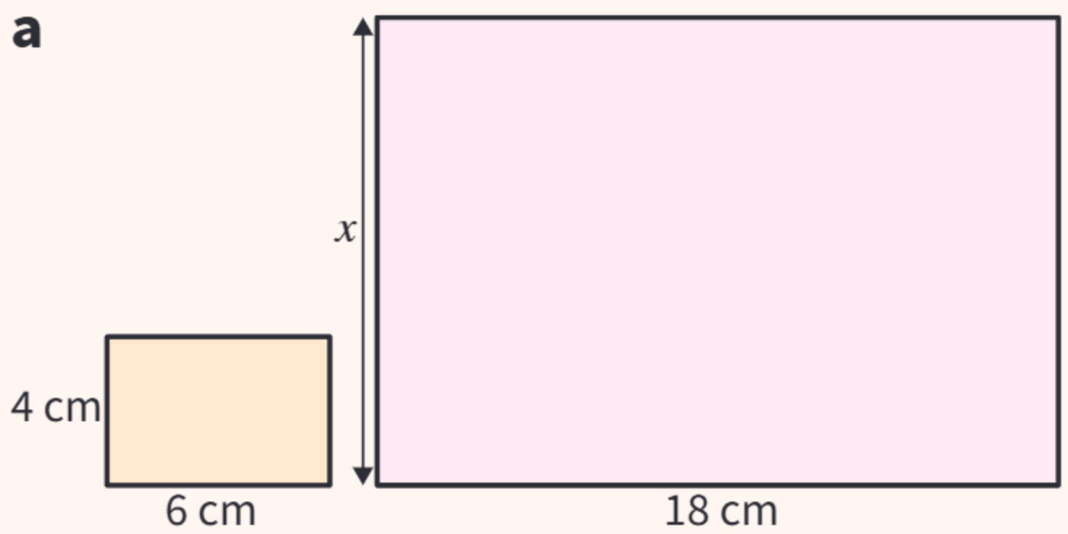
c z.

1.2 Intelligent practice

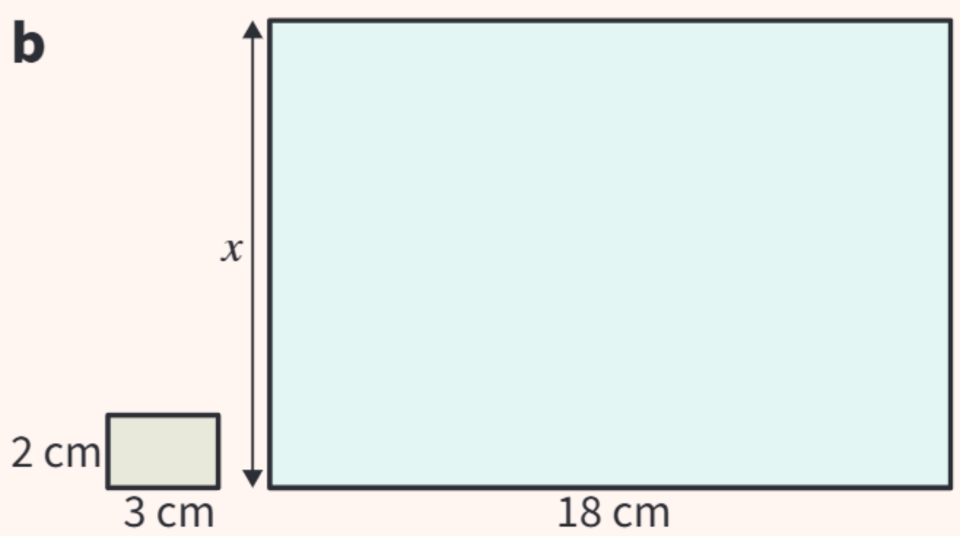
In each question, you might notice something when you move from one question part to the next. What is different between each question part (e.g. **1b**) and the one that came before (e.g. **1a**)? Decide how you expect the answer to be different. Then work through the question and check your answer. Think about why your prediction was right or wrong.

1 For each pair of similar rectangles, find the scale factor and the length marked x.

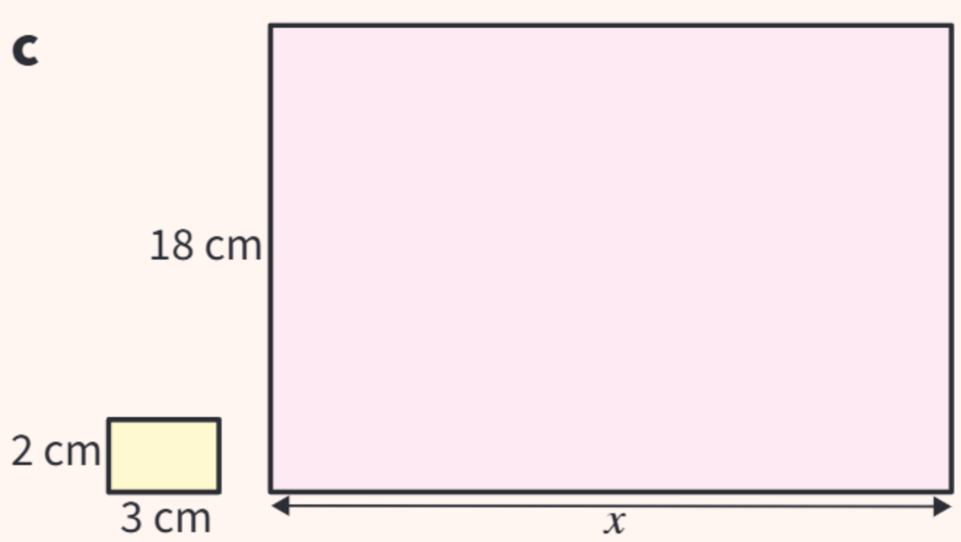
a



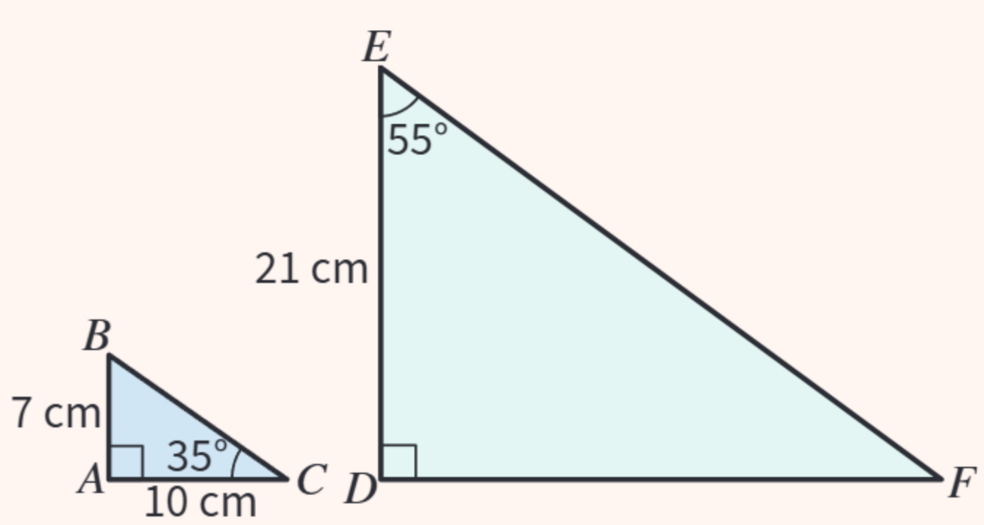
b



c



2 Triangle ABC is similar to triangle DEF.



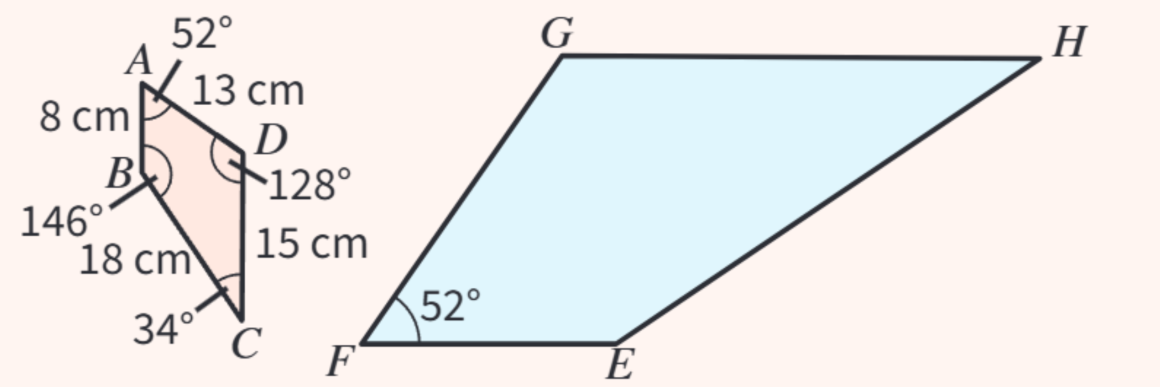
a Write down the value of:

i $E\hat{F}D$ **ii** $A\hat{B}C$.

b What is the scale factor for the enlargement from triangle ABC to triangle DEF?

c Find the length of DF.

3 $ABCD$ is similar to $EFGH$. The scale factor of the enlargement from $ABCD$ to $EFGH$ is 4.



Write the value of:

a $\angle GHE$

b FE

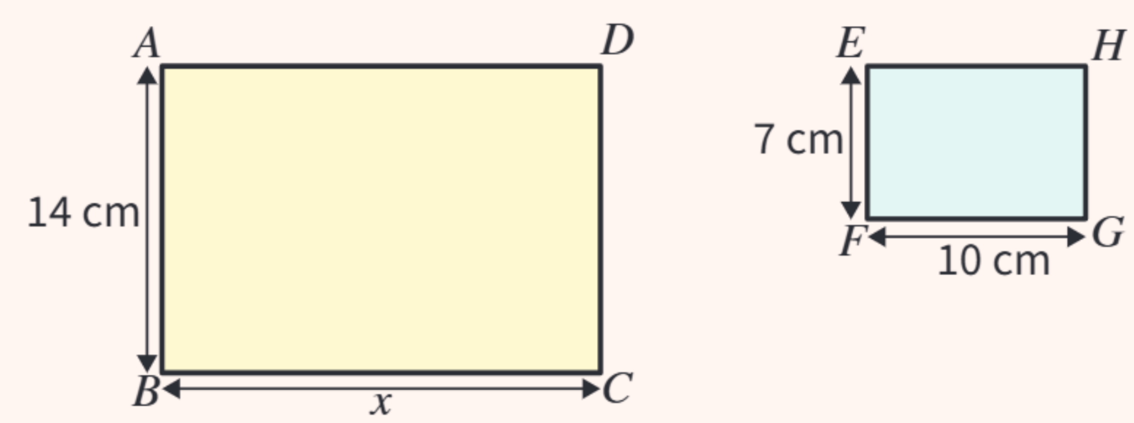
c $F\hat{G}H$

d GH.

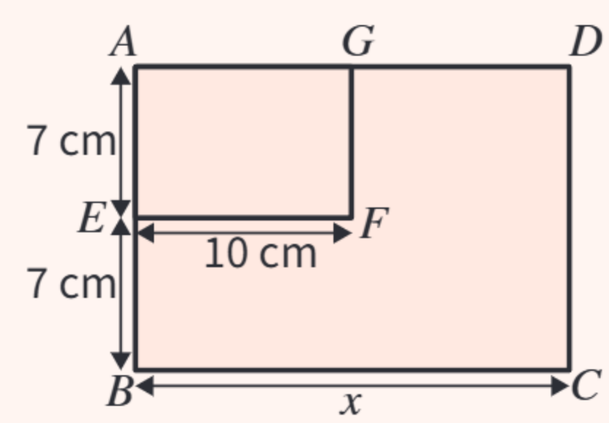
e EH.

4 Find the length marked x. Give your answers to 1 d.p. where appropriate.

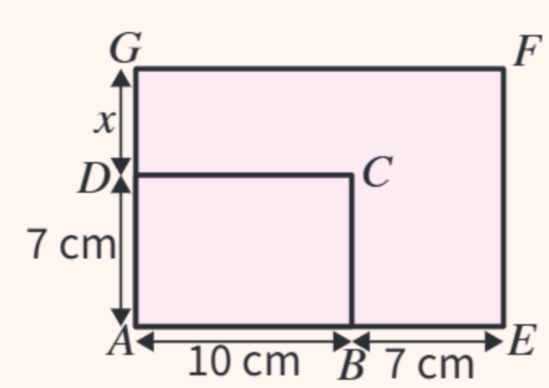
a $ABCD$ and $EFGH$ are similar.



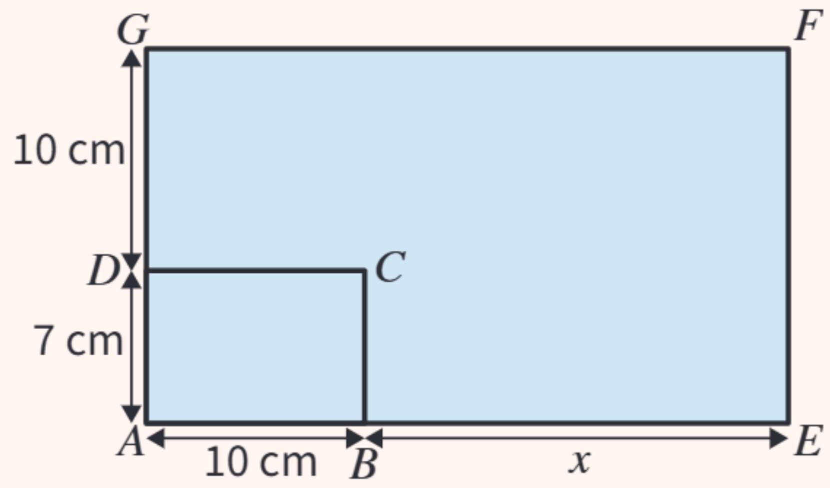
b $ABCD$ and $AEFG$ are similar.



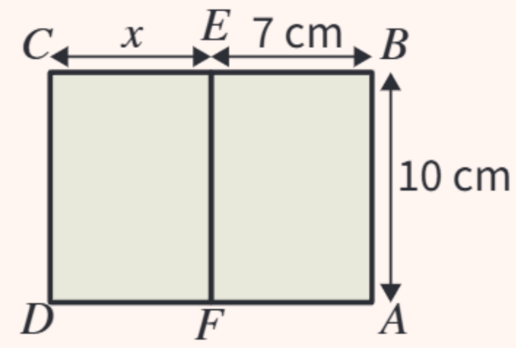
c $ABCD$ and $AEFG$ are similar.



d *ABCD* and *AEFG* are similar.



e *ABCD* and *ABEF* are similar.

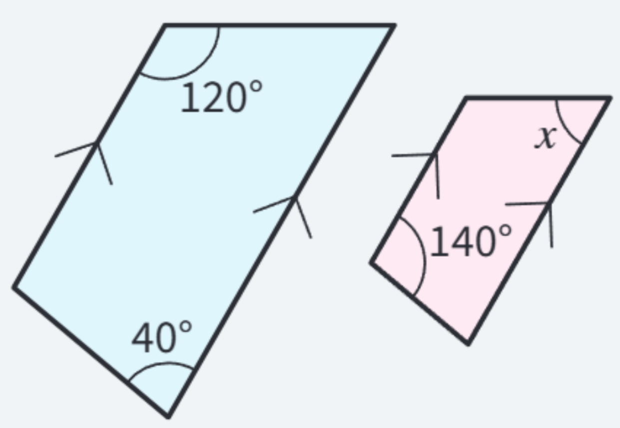


1.2 Which method?

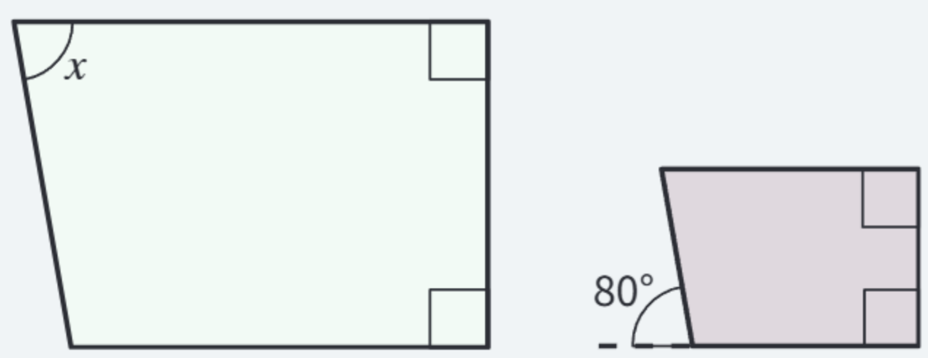
In these questions, you will need to think carefully about which methods to apply.
For some questions, you might need to use skills from Student Book 7 or Student Book 8.

1 The shapes are similar. Find the value of x.

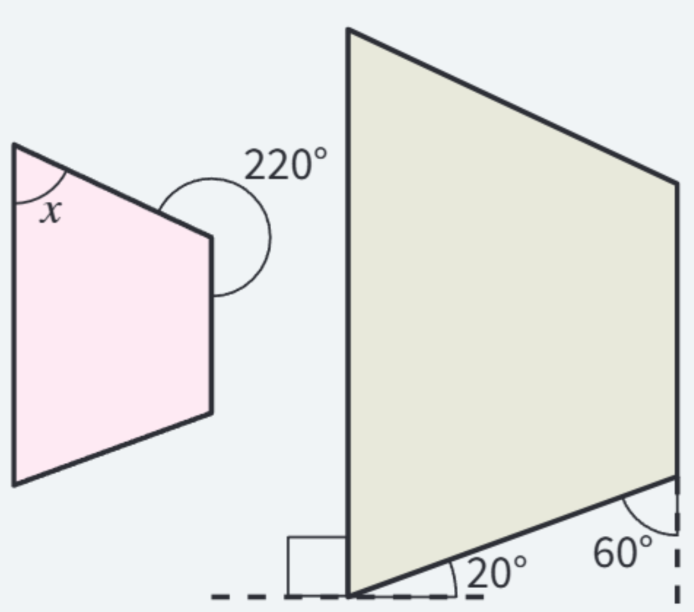
a



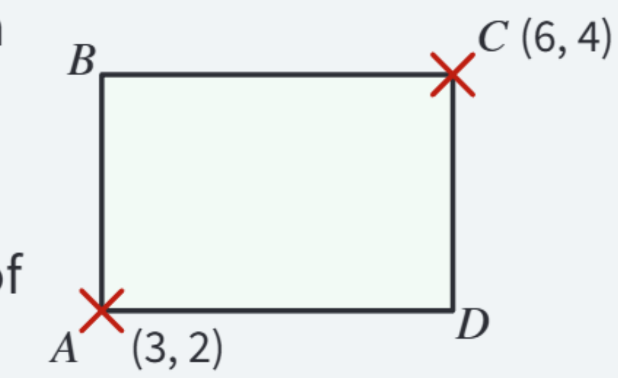
b



c



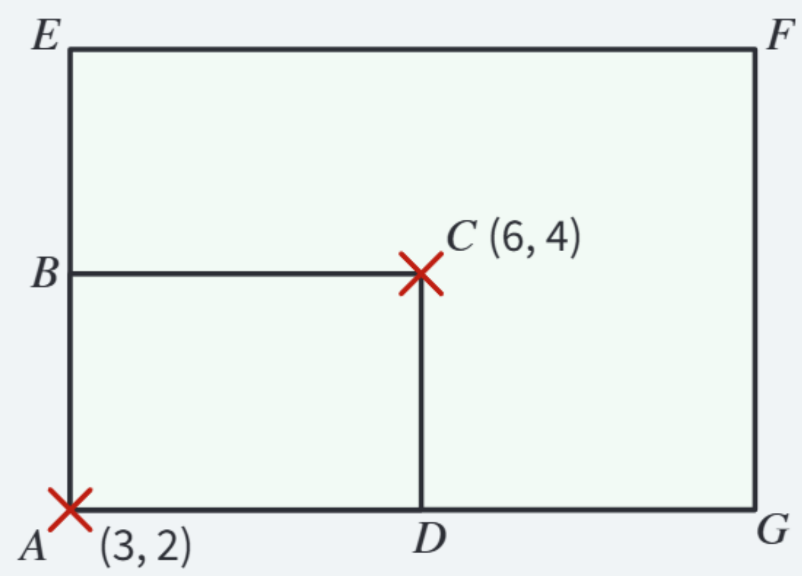
2 *ABCD* is a rectangle with coordinates for points *A* and *C* shown.



a Find the coordinates of points *B* and *D*.

b Find the perimeter of *ABCD*.

3 *ABCD* has been enlarged to rectangle *AEFG*. Both rectangles are shown on the diagram below.



What is the scale factor of enlargement if:

a $AD = DG$

b $AD : DG = 1 : 4$

c E is at $(3, 10)$

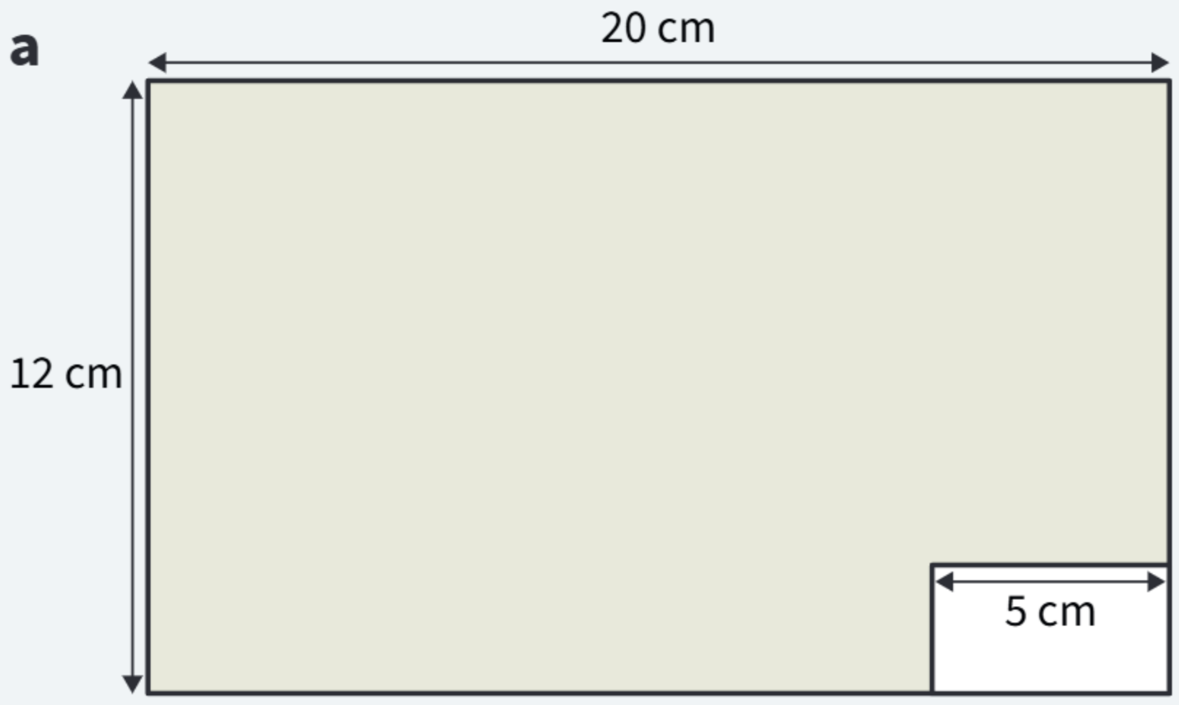
d F is at $(7.5, 5)$

e $GF = 12$

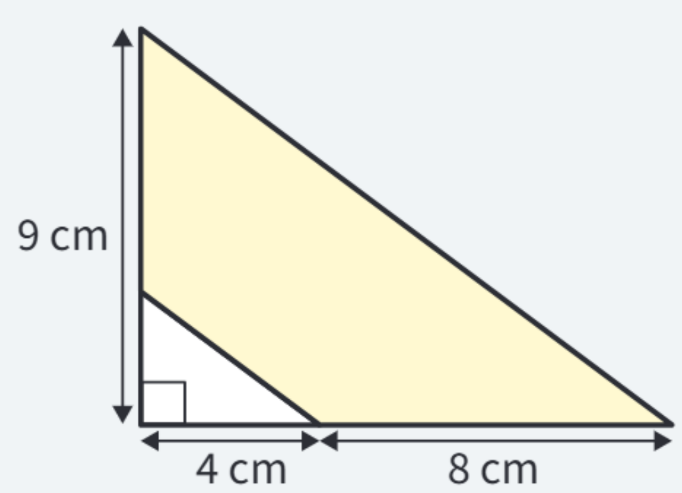
f $EF = 12$?

4 Each shape is made of similar polygons. Find the shaded (darker) areas.

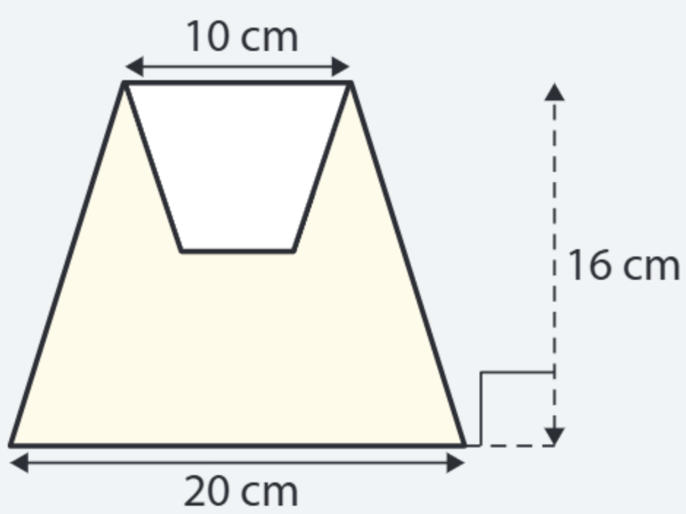
a



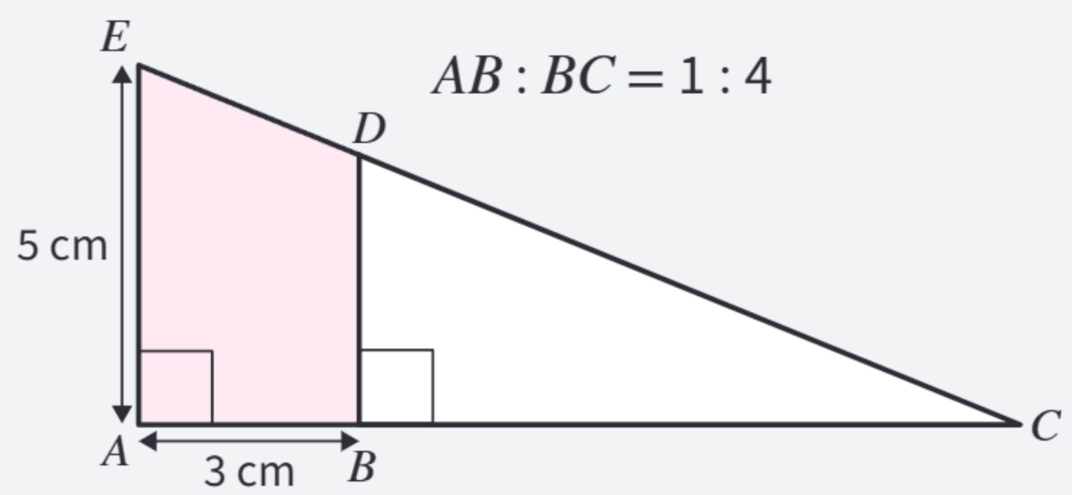
b



c



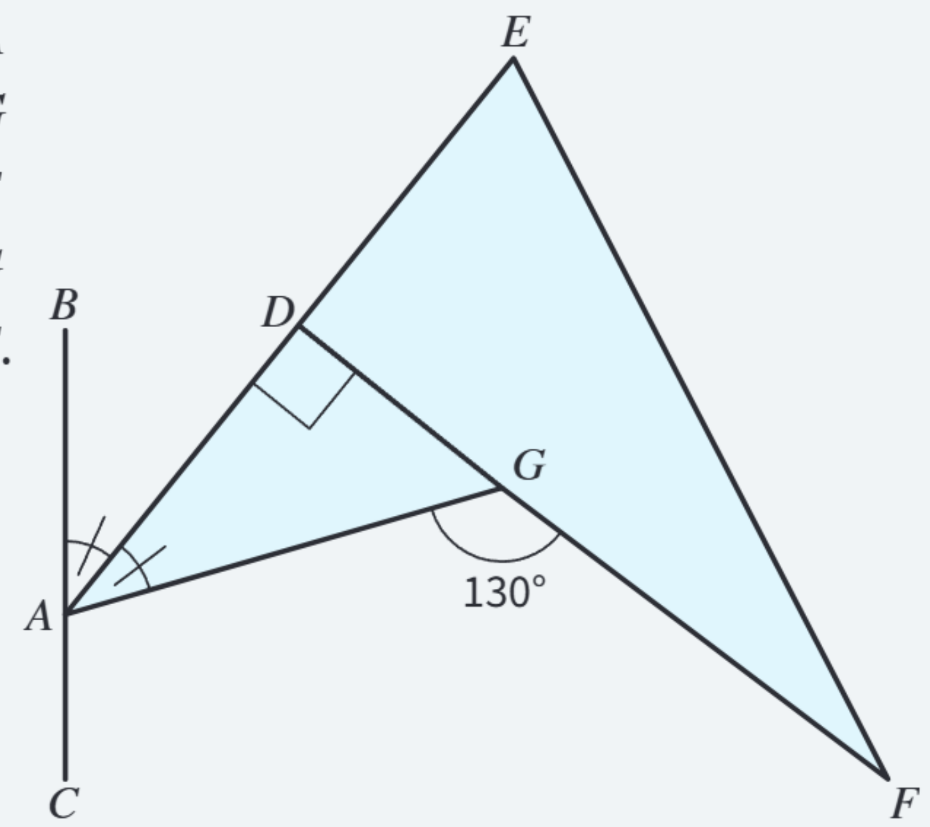
d



5 Triangle ADG is similar to triangle DEF.

Find the value of angle:

a $D\hat{G}A$
b $D\hat{A}G$
c $D\hat{E}F$
d $C\hat{A}G$
e $D\hat{F}E$.



6 Triangle ABC is similar to triangle DAC.

Find the value of angle:

a $A\hat{D}C$
b $D\hat{C}B$.